Applicability of the microwave cavity perturbation method for conductivity measurements on carbon fibers

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A microwave cavity perturbation technique has been used to measure the resistance per unit length of carbon fibers. These materials and other test materials used in these measurements such as copper, evenohm, and tungsten, have much higher conductivities than polymeric materials which have been investigated previously with this technique. The 1500–5000-Ω resistance of the carbon fibers lay in the region near the limit of applicability of the perturbation technique, thereby affording a means of investigating this limit. The condition $R_s/Z_c > 2$ where R_s is the sample resistance and Z_c is the characteristic impedance of the cavity, appears to be a useful condition for estimating the limit of applicability.

I. INTRODUCTION

Contactless resistivity techniques are useful for several different reasons. Such methods avoid damage and property modification to the sample that can result from attaching leads, and can be used in situations involving extreme conditions of pressure, temperature, or corrosive environments. A recent overview of techniques has been given by McCrae *et al.*[1]

A microwave technique for measuring the resistance per unit length of highly conducting materials in the form of fibers, whiskers, or thin wires would be extremely useful. Delaney and Pippard[2] have reviewed microwave techniques for resistivity measurements, while cavity perturbation methods for measuring the resistance per unit length of wires or filaments with specific conductivity $\sigma \leqslant 10^4\ (\Omega\ \mathrm{m})^{-1}$ have been discussed by Buranov and Schegolev[3] and Khanna *et al.*[4]

Microwave methods are very useful for the measurement of the conductivity of rods of polymeric or other low conductivity materials, since the excited electric current flows parallel to the axis of the rod. The other principal noncontact method, the inductance method, excites ring currents flowing perpendicular to the axis.[1] The planar conductivity is usually of less technological interest than the axial conductivity. Furthermore, it would be difficult to apply the inductance method to fibers or wires of diameter only $\lesssim 10$ μm, since frequencies as high as 500 MHz would be required.

The purpose of this brief note is to report some recent microwave measurements made on more highly conducting materials. The results provide some guidelines concerning the limits of applicability of the technique for conducting samples.

II. EXPERIMENT

It is well known[5,6] that if a sample is inserted into a resonant cavity then the resonant frequency is shifted due to the real part of the dielectric constant of the sample, while the quality factor Q of the cavity is reduced due to the imaginary part. This latter effect can be utilized to measure the resistance per unit length of the sample. The sample, in the form of a cylinder, is inserted into a TE_{102} rectangular cavity, as shown in Fig. 1. If the sample has sufficiently high resistance, then it can be inserted at the position of maximum electric field. A measurement of the change in the Q of the cavity relative to the empty cavity Q can then be used to determine the resistance of the sample. However, if the sample resistance is smaller than a critical value (e.g., ~ 100 kΩ for the cavity discussed), then the electric field at the fiber location is reduced to close to zero, the cavity mode becomes substantially changed from the TE_{102} mode, and the method becomes impractical.

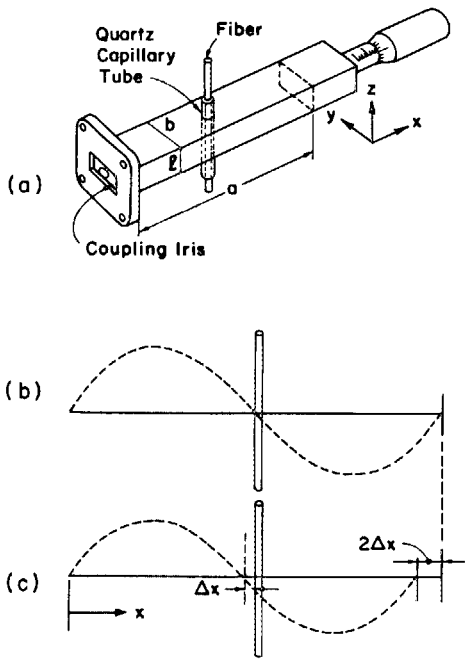

FIG. 1. Sketch of the method used for the cavity perturbation technique: (a) the cavity, with a moveable end piston, (b) the variation of electric field for TE_{102} mode with sample in the zero field position, (c) the end piston moved so that sample is moved from zero-field position.

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TABLE I. Ratios for sample resistance to cavity impedance (R_s/Z_c)($Z_c = 500\,\Omega$).

Sample	35 MSi fiber	85 MSi fiber	Tungsten	Evenohm	Copper
Diameter (μm)	6.3 ± 0.2	5.7 ± 0.3	12.7 ± 0.2	76 ± 0.5	76 ± 0.5
$R_s(\Omega)$	4800 ± 7	1001 ± 2	4.80 ± 0.06	2.75 ± 0.03	$(37.3 \pm 0.2)\times 10^{-3}$
R_s/Z_c	10	2	0.01	5×10^{-3}	7×10^{-5}
$(R/l)_{dc}/(R/l)_{\mu wave}$	1.00 ± 0.04[a]	0.78 ± 0.03[a]	$(2.0 \pm 0.3)\times 10^{-3}$[a]	$(5.4 \pm 0.5)\times 10^{-4}$[a]	$(1.0 \pm 0.2)\times 10^{-4}$[a]
	1.05 ± 0.05[b]	1.05 ± 0.04[b]	$(5.1 \pm 0.3)\times 10^{-3}$[b]	$(1.7 \pm 0.4)\times 10^{-3}$[b]	$(2.9 \pm 0.6)\times 10^{-3}$[b]

[a] Without skin depth correction.
[b] With skin depth correction based on dc resistivity.

An alternative approach for moderately conducting fibers is to place the sample near the zero electric field position. If the sample has a small volume then Q remains essentially unchanged if it is inserted exactly into the zero field position. Then, by changing the position of the adjustable short at the end of the cavity so that the sample "sees" a higher electric field strength, the cavity will be loaded to successively higher values. The corresponding changes in the Q of the cavity can then be used to calculate the sample resistance per unit length.

The perturbation approach assumes that the sample does not change the electric field in the cavity. This assumption clearly breaks down for good conductors, since the electric field must be zero at the surface of a perfect conductor. Within the perturbation approximation the resistance per unit length R/l of the conductor is given by

$$\frac{R}{l} = \frac{4l_s \sin^2(K\Delta x)}{\epsilon_0 \omega V_c \delta(1/Q)}. \tag{1}$$

In Eq. (1), l_s is the effective length of the sample in the cavity, K is $(\omega^2 - \omega_c^2)/c$, ϵ_0 is the permitivity of free space, V_c is the volume of the cavity, and $\delta(1/Q)$ is given by $(1/Q_{\text{sample}} - 1/Q_0)$. In the above, ω is the excitation frequency and ω_c is the cut-off waveguide frequency $\pi c/b$, for a cavity width b (TE_{102} mode), and speed of light c. Q_{sample} is the loaded Q of the cavity with the sample for the short displaced Δx from the position corresponding to zero electric field at the fiber position, and Q_0 is the Q of the cavity for $\Delta x = 0$.

The measurements were carried out at 10 GHz, on thin wires of copper, evenohm, and tungsten and on carbon fibers. The relaxation frequency is much higher than 10 GHz for the conduction processes involved in these materials, so that the conductivity is essentially the same as the dc conductivity. The resistivity of the metals is such that the skin depth Δ at 10 GHz is small compared to the wire diameter d. Hence, a large correction is needed to obtain the true resistance per unit length. For the carbon fibers, d and Δ are comparable so that the correction is relatively small.

Results comparing the resistance per unit length obtained from the microwave and a four-point dc technique are listed in Table I and shown in Fig. 2. It can be seen that the microwave technique gives the correct value for R/l for values above $\sim 2\times 10^5\,\Omega\,\text{m}^{-1}$ but increasingly breaks down for smaller values.

When the length of the copper wire became comparable to that of the width of the cavity, the uncertainty in the experimental value of $\delta(1/Q)$ increased. Values of $\delta(1/Q)$ depended strongly on the exact position of the wire in the cavity. Furthermore, when the wire length exceeded that of the cavity, $\delta(1/Q)$ continued to decrease. This is evidence for antenna radiation. The effect was less important for evenohm and tungsten wires, and could not be detected for the carbon fibers tested. Further evidence for the breakdown of the perturbation formula for the metallic wires arose from the variation of $\delta(1/Q)$ with sample length, chosen to be smaller than that of the cavity. For carbon fibers, $\delta(1/Q)$ varied linearly with fiber length, ($l < b$), as predicted by Eq. (1), but dropped more rapidly than linearly for the metallic wires.

III. VALIDITY OF THE PERTURBATION FORMULA

Several authors have pointed out the limitations of the cavity perturbation technique. Spencer *et al.*[7] suggest that a sufficient criterion for the use of Eq. (1) could be that the electric field outside and inside the sample are approximately equal, or that the change in the overall geometrical configuration of the fields be small when the sample is inserted in the cavity. Bonincontro and Cametti[8] show that a necessary condition is that $\delta(1/Q) \lesssim (f_0 - f_s)/f_0$, where f_0 and f_s are the resonant frequencies of the cavity without the sample and with it, respectively. This gives an upper limit for the change in Q which is allowed for Eq. (1) to be applicable. However, the first of these conditions is too general in nature to be of

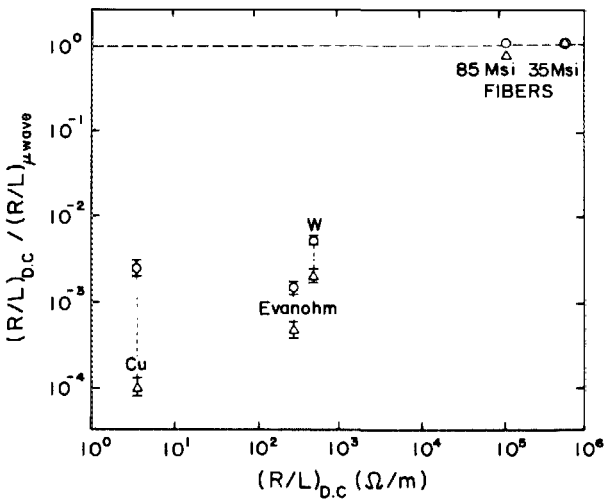

FIG. 2. Variation of the ratio of the resistance per unit length measured by the dc method, (R/L) to the microwave value, $(R/L)_{\mu wave}$, as a function of the (R/L): △–without skin depth correction; O–with skin depth correction using dc resistivity.

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practical interest, while it was found from the present series of measurements that the second condition was also not of practical use for deciding whether Eq. (1) was applicable or not.

Vivmanec[9] has derived the equivalent circuit impedance based on the cavity perturbation theory and shows that the practical requirement for the validity of the method can be expressed as

$$R_s > 2Z_c = \frac{2\eta}{\sqrt{1-\omega_c/\omega}}, \tag{2}$$

where R_c is the sample resistance, Z_c is the impedance of the cavity, and η is the intrinsic impedance of the medium, approximately 376 Ω for free space. Values for the ratio R_s/Z_c are given in Table I for the experiments conducted. It can be seen that the validity criterion is well satisfied for the 35 MSi fiber (Young's modulus $\sim$240 GPa), is marginally satisfied for the 85 Msi fiber and is invalid for the wires.

IV. CONCLUSION

The cavity perturbation technique for measuring the resistance per unit length of filamental conductors has been examined experimentally for samples of higher conductivity than those previously reported. The method breaks down when the sample resistance shorting the cavity R becomes less than about twice the characteristic impedance of the cavity Z_c. This limitation severely curtails the use of the cavity perturbation technique for highly conducting materials such as intercalated fibers.

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