Spin-wave instability theory for hexagonal ferrites[a)]

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The spin-wave instability theory for ferrites with magnetocrystalline anisotropy is extended to include planar anisotropy and second-order processes. General expressions are obtained for first- and second-order processes in planar hexagonal ferrites magnetized in the easy plane and pumped with an arbitrary microwave field configuration. The calculations for second-order processes predict characteristic curves for the critical microwave instability threshold field versus the static magnetic field. These curves, with a minimum at ferromagnetic resonance, are similar to the butterfly curves encountered in the analysis of subsidiary absorption in cubic crystals. The results are applied to previously published experimental data on Zn_2Y-type hexagonal ferrite platelets.

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INTRODUCTION

Hexagonal ferrites with planar anisotropy have been shown to exhibit peculiar nonlinear behavior[1-3] due to the large anisotropy field present in many of these materials. This large anisotropy field shifts the spin-wave manifold upward in frequency, so that the observed nonlinear effects often are due to second-order processes, even at x-band frequencies. The general theory for spin-wave instabilities was developed by Suhl,[4] Schlömann,[5] and Morgenthaler.[6] The effect of anisotropy on spin-wave dispersion and instability processes was considered by several authors.[7-12] In this work the approach of Schlömann[5] is extended to include planar anisotropy, based on the effective field formulation by Patton.[12] The results are applied to the previously published experimental data on Zn_2Y.[3] The dependence of the second-order instability threshold on applied static field is examined for fields near ferromagnetic resonance (FMR).

SPIN-WAVE DISPERSION RELATION

Consider an ellipsoidal shaped single crystal magnetized along one principal axis by an external static magnetic field $\bar{H}_0$ as shown in Fig. 1(a). The XYZ coordinate system with Z axis along the static field direction and X and Y along the other two principal axes of the ellipsoid defines the customary spin-wave coordinate system in which the direction of the spin-wave wave vector $\bar{k}$ is specified by the usual polar and aximuthal angles θ_k and ϕ_k. The crystallographic coordinate system $X'Y'Z'$ is taken such that the Z' axis is parallel to the hexagonal c axis of the crystal. The relative orientation of these two coordinate systems is shown in Fig. 1(b). Here, the angles θ and ϕ define the direction of the field $\bar{H}_0$ in the $X'Y'Z'$ system. For convenience, ϕ is set equal to zero in the analysis to follow. We assume that the magnetization $\bar{M}$ is composed of a static component parallel to $\bar{H}_0$ and a dynamic magnetization $\bar{m}$ with $|\bar{m}| \ll M$. The magnetization vector $\bar{M}$ can be written in terms of a unit vector $\bar{\alpha}$ such that $\bar{M} = M\bar{\alpha}$. The expansion of $\bar{\alpha}$ in a Fourier series makes it possible to obtain a formulation in terms of α_k, corresponding to the classical amplitude of a general spin wave with wave vector $\bar{k}$. The formulation is based on the classical equation of motion for $\bar{M}$ and the effective field approach of Ref. 12. The total effective field $\bar{H}_{\text{eff}}$ can be written as

$$\bar{H}_{\text{eff}} = \bar{H}_0 + \bar{h}_0 + \bar{H}_{\text{dipolar}} + \bar{H}_{\text{exchange}} + \bar{H}_{\text{anisotropy}}, \quad (1)$$

where $\bar{h}_0$ is the applied microwave field, and $\bar{H}_{\text{dipolar}}$ and $\bar{H}_{\text{exchange}}$ are fields due to the dipole-dipole interaction and the exchange interaction, respectively, and are the same as those given in Ref. 12. The effective anisotropy field $\bar{H}_A$ consists of a static component $\bar{H}_A$ and a dynamic component $\bar{h}_A$. These can be obtained from the magnetocrystalline anisotropy energy, which for the easy plane hexagonal ferrites is given to the first order by[13]

$$F_A = K_1(1 - \alpha_3^2) = K_1(\alpha_1^2 + \alpha_2^2), \quad (2)$$

where K_1 is the first order anisotropy constant and $\alpha_1, \alpha_2, \alpha_3$ are the direction cosines of $\bar{M}$ in $X'Y'Z'$ system. From the $(1 - \alpha_3^2)$ form of the anisotropy, the static component of the anisotropy field is

$$\bar{H}'_A = -\nabla_\alpha F_A / M = (2K_1/M)\alpha_3 \hat{z}', \quad (3)$$

where ∇_α denotes gradient with respect to the direction cosines and $\hat{z}'$ a unit vector along the Z' axis. The dynamic effective anisotropy field $\bar{h}'_A$ is obtained by evaluating the variation of $\bar{H}'_A$ with respect to the direction cosines.

$$\bar{h}'_A = \frac{2K_1}{M}\begin{pmatrix} 0 \\ 0 \\ \delta\alpha_3 \end{pmatrix} = \frac{2K_1}{M^2}\begin{pmatrix} 0 \\ 0 \\ m_{z'} \end{pmatrix}. \quad (4)$$

When these expresisons are transformed to the XYZ spin-wave system, the fields become

$$\bar{H}_A = \frac{2L_1}{M}\begin{pmatrix} -\sin\theta\cos\theta \\ 0 \\ \cos^2\theta \end{pmatrix}, \quad (5)$$

and

$$\bar{h}_A = \frac{2K_1}{M^2}\begin{pmatrix} \sin^2\theta & 0 & -\cos\theta\sin\theta \\ 0 & 0 & 0 \\ -\cos\theta\sin\theta & 0 & \cos^2\theta \end{pmatrix}\begin{pmatrix} m_x \\ m_y \\ m_z \end{pmatrix} = \mathrm{A}\bar{m}. \quad (6)$$

It is to be noted that, even though the $(\alpha_1^2 + \alpha_2^2)$ form of the anisotropy in Eq. (2) would lead to different expressions for $\bar{H}_A$ and H_A, they yield identical results for spin-wave disper-

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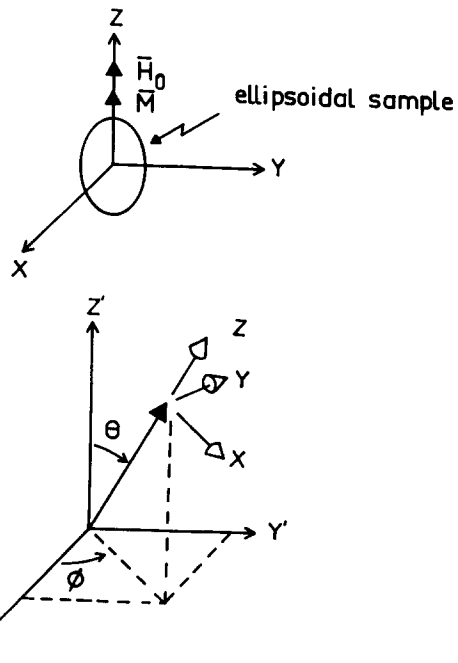

FIG. 1. (a) Sample orientation in the spin-wave coordinate system XYZ. (b) Relative orientation of spin-wave coordinate system with respect to crystallographic coordinate system $X'Y'Z'$. The Z' axis is taken parallel to the uniaxial direction.

sion, etc.

The magnetocrystalline anisotropy enters into the theory through the components of $\bar{H}_A$ in Eq. (5) and of the A matrix in Eq. (6). The general spin-wave dispersion relation is[12]

$$\omega_k^2 = (A_k^0 + B_k^0 - \gamma MA_{xx})(A_k^0 - B_k^0 - \gamma MA_{yy}) - 2\gamma MA_B_k^0 \sin^2\theta_k. \tag{7}$$

The various parameters in Eq. (7) are defined in Ref. 12. They may be evaluated for the case of planar anisotropy, using Eqs. (5) and (6). The resulting general spin-wave dispersion relation for a planar hexagonal ferrite is

$$(\omega_k/\gamma)^2 = (H - H_a\cos^2\theta + Dk^2 + 4\pi M\sin^2\theta_k + H_a\sin^2\theta) \times (H - H_a\cos^2\theta + Dk^2) + 4\pi MH_a\sin^2\theta\sin^2\theta_k\sin^2\phi_k, \tag{8}$$

where the replacements $H = H_0 - 4\pi MN_z$ and $H_a = -2K_1/M$ have been made, and γ is the gyromagnetic ration (taken as positive). For the particular case of $\theta = \pi/0.2$, i.e., a sample magnetized in the easy plane, the dispersion relation reduces to

$$(\omega_k/\gamma)^2 = (H + Dk^2 + 4\pi M\sin^2\theta_k + H_a)(H + Dk^2) - 4\pi MH_a\sin^2\theta_k\sin^2\phi_k. \tag{9}$$

in agreement with the expressions obtained by Schlömann *et al.*[14] and Mita.[2]

EVALUATION OF INSTABILITY PROCESSES

To find the various instability thresholds the basic approach is to calculate a coupling parameter G_k^a for the power coupled in the spin system, and

$$G_k^a = \sum_{n=-2}^{+2} G_k^{a(n)} e^{i\omega nt}. \tag{10}$$

The various order instability thresholds are obtained by simply equating the corresponding $G_k^{a(n)}$ to the spin-wave relaxation rate η_k. The harmonic terms in G_k^a are the result of different combinations of the uniform precession amplitude α_0 and its complex conjugate α_0^*. These can be separated conveniently by writing α_0 in terms of α_{L0} and α_{A0}, complex time independent coefficients characterizing the Larmor and anti-Larmor parts of the uniform precession, respectively.

$$\alpha_0 = \alpha_{L0}e^{i\omega t} + \alpha_{A0}e^{-i\omega t}. \tag{11}$$

General expressions for α_{L0} and α_{A0} for the case of no damping are given in Ref. 15. This uniform precession mode is driven by the transverse components of the applied microwave field. Taking this field as

$$h_0^{x,y} = h_0 a_{x,y}\cos(\omega t + \delta_{x,y}), \tag{12}$$

and

$$h_0^z = h_0 a_z\cos(\omega t), \tag{13}$$

appropriate combinations of the terms a_x, a_y, a_z, and δ_x and δ_y may be used to rise to obtain specific microwave polarization configurations.

For the first order process, corresponding to the $n = 1$ term in Eq. (10) and the pumping of spin-waves at half the external microwave frequency ($\omega_k = \omega/2$), the instability thresholds can be obtained by using Eqs. (60)–(65) in Ref. 12 with the parameters A_k^a, B_k^a, ω_{zx}, ϕ_a, etc., evaluated for planar anisotropy using the elements of matrix A defined in Eq. (6) above. Since, in the cases of most interest for planar hexagonal ferrite, the instability process occurs near resonance, the Larmor and anti-Larmor precession amplitudes in Ref. 15 must be modified to include damping.

For nonspherical samples, at X band the parametric excitation of spin waves at half the pump frequency is often forbidden. In this case, second order processes corresponding to the $n = 2$ term in Eq. (10) must be considered. The unstable spin waves for such second order processes will have the same frequency as the microwave field; i.e., $\omega_k = \omega$. Analysis of the $n = 2$ terms in $G_k^{(a)}$ yields

$$G_k^{a(2)} = \frac{e^{2i\phi_a}}{2\omega_k}\left\{\frac{e^{-2i\phi_a}}{2}(A_k^a + \omega_k)(q_L^2 c_k + q_L q_A^* d_k + q_L a) + \frac{e^{2i\phi_a}}{2}(A_k^a - \omega_k)(q_A^{*2} c_k + q_L q_A^{*2} d_k^* + q_A^* a^*) - B_k^a\left[q_L q_A^* p + (q_L^2 + q_A^{*2})r + \frac{1}{2}(q_L a + q_A^* a^*)\right]\right\}, \tag{14}$$

where q_L and q_A are defined through $\alpha_{L0} = (\gamma h_0 2)q_L$ and $\alpha_{A0} = (\gamma h_0/2)q_A$ and the coefficients c_k, d_k, p, and r are as given below.

$$c_k = \gamma MA_+/2 - \gamma MA_{zz} + \gamma Dk^2 + \omega_m[\cos^2\theta_k - (N_x + N_y)/2]. \tag{15}$$

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$$d_k = [2\gamma MA_- - \omega_m(\sin^2\theta_k e^{2i\phi_k} + N_x - N_y)]/2. \quad (16)$$

$$p = \omega_m[2N_z + 3\cos^2\theta_k - 1 - (N_x + N_y) + \gamma MA_- - \gamma MA_{zz}]/2. \quad (17)$$

$$r = [\gamma MA_- - \omega_m(N_x - N_y)]/4. \quad (18)$$

The parameters ω_m and a are given by $\omega_m = \gamma 4\pi M$ and $a = a_x + ia_y$. Since the various G_k^a terms correspond to the power coupled into the spin system, the instability condition $|G_k^{a(2)}| = \eta_k$ leads directly to an analytical result for the spin-wave instability threshold h_c for second order processes. Expressed in terms of the usual spin-wave linewidth, $\Delta H_k = 2\eta_k/\gamma$, the critical microwave field amplitude for spin-wave instability is

$$h_c = 2\sqrt{\Delta H_k \omega/\gamma W}, \quad (19)$$

where W is defined through

$$G_k^{a(2)} = \exp(2i\phi_a)(\gamma h_0/2)^2(W/2\omega). \quad (20)$$

In the case of a linearly polarized transverse microwave field and the static magnetic field at FMR, the present analysis is in agreement with the results of Schlömann *et al.*[14]

APPLICATION TO Zn_2Y

As indicated in the introduction, part of the motivation for the present theoretical extension of spin-wave instability processes to hexagonal ferrite was the peculiar FMR effects in Zn_2-Y.[3] For the parameters corresponding to Zn_2Y and the experiment in Ref. 3, namely, $N_x = 0.6$, $N_y = N_z = 0.2$, $H_a = 9$ kOe, $4\pi M = 2.85$ kG, and $\omega/2\pi = 8.9$ GHz, first order instability processes are precluded for static fields on the order of the field required for FMR. Instability effects should be limited to second order processes. The result given above for $G_k^{a(2)}$ was therefore applied to calculate threshold fields and spin-wave linewidth for Zn_2Y biased at or near FMR using the parameters given above and an FMR linewidth $\Delta H = 40$ Oe. The calculation was done for the case of transverse pumping with a linearly polarized microwave field along the y direction. At any given field, the allowed values of θ_k and ϕ_k were scanned in one degree steps in order to determine the lowest threshold. The result is shown in Fig. 2 where the minimum threshold microwave field amplitude is plotted versus the static field. The curve is similar in shape to the butterfly curves encountered in the analysis of subsidiary absorption in cubic crystals for first order processes.[15] The minimum for this second-order butterfly curve occurs at the FMR field, corresponding to a maximum in the Larmor precession amplitude at resonance. The spin waves for which the minimum thresholds in Fig. 1 are realized propagate at $\theta_k = 90°$, perpendicular to the applied static field direction. This contrasts rather sharply with the result for second-order processes in isotropic materials, $\theta_k = 0°$.[5] Presumably the high degree of ellipticity for the spin precession in planar anisotropy materials results in a much stronger coupling at $\theta_k = 90°$.

Based on the above calculations, it is possible to make numerical estimates of the spin-wave linewidth in Zn_2-Y from the observed microwave field amplitudes for foldover or nonlinear behavior. The recent data of Silber *et al.* in Ref. 3 indicate very low threshold amplitudes of 15–30 mOe, corresponding to spin-wave linewidths of 1.5–6 mOe. These linewidths seem to be unrealistically small. The data in Ref. 3 were obtained, however, with cw microwave excitation. Although heating effects were apparently eliminated as a possible cause of the foldover behavior by comparing results for different heat sinking configurations for the Zn_2-Y platelet, recent measurements (as yet unpublished) do indicate that the apparent threshold for foldover increases as the duty cycle for the incident microwaves is decreased from 100%. This is consistent with previous results on hexagonal ferrite obtained for very low duty cycles. Savage *et al.*[16] have reported a spin-wave linewidth of 0.74 Oe for MnZn-Y under parallel pumping at 17 GHz. Based on the present theory, thresholds reported by Mita[2] for transverse pumping in Zn_2:Yspheres at 9 GHz yield a spin-wave linewidth of 1.2 Oe. These linewidths appear to be much more reasonable. Further work is in progress to determine the physical processes behind the peculiar low power distortions observed by Silber *et al.*,[3] to determine the effect of duty cycle on such processes, and to characterize the wave vector dependence of ΔH_k in hexagonal ferrite.

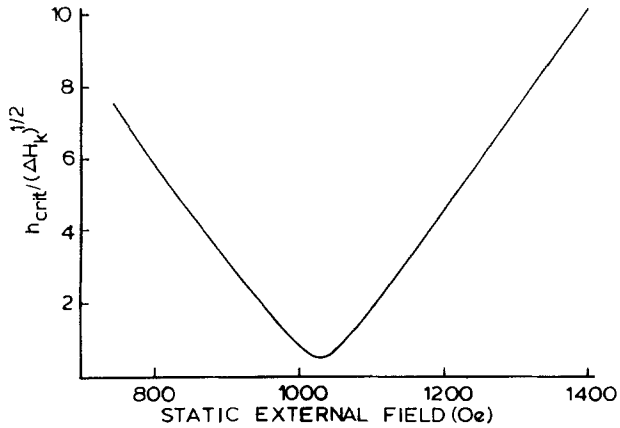

FIG. 2. Second-order spin-wave instability threshold field h_{crit} normalized to the spin-wave linewidth ΔH_k as a function of static external field. The curve was obtained for Zn_2Y platelet with appropriate material parameters from the literature[5] and experimental parameters corresponding to the experimental situation in Ref. 3.

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