Simple analytic method for microwave cavity *Q* determination

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A simple method for the accurate determination of the quality factor, Q, of a reflection microwave cavity is described.

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INTRODUCTION

The accurate determination of the quality factor, often called the Q factor or simply Q, of microwave cavities is often tedious and limited in accuracy. The textbook procedure[1,2] generally involves the measurement of voltage standing wave ratio (VSWR) or reflection coefficient vs. frequency over a range of frequencies which straddles the cavity resonance frequency ω_r. Graphical data analysis or iterative procedures are then used to determine the frequency half-power points, and hence the cavity Q. This article describes a very simple method to determine the Q which does not involve either of these procedures. Rather, the measurements of reflection coefficient vs. frequency are used to determine a span of a few MHz which straddles the cavity resonance. The three Q parameters are easily determined from a straightforward linear least-square fitting procedure. The technique is an adaptation of the basic idea discussed by Frait[3] in the Czech language literature of 1959. It is similar, in principle, to the "straight line" method discussed in Ref. 2.

I. ANALYTICAL METHOD

The method is based on the observation that the cavity voltage reflection coefficient ρ, defined by

$$\rho = V_{0R}/V_{0I}, \tag{1}$$

where V_{0I} is the amplitude of the incident voltage in a transmission line and V_{0R} is the amplitude of the reflected voltage, may be expressed in terms of the line impedance Z_0 and the cavity impedance Z_C,[1]

$$\rho = \frac{V_{0R}}{V_{0I}} = \frac{|Z_C - Z_0|}{Z_C + Z_0}\,. \tag{2}$$

The circuit model for a cavity with an inductive iris is given in Fig. 1, with V: drive amplitude; Z_0: line impedance; L_a: Iris inductance; n: iris coupling coefficient; R, L, C: equivalent circuit cavity resistance, inductance, and capacitance. Figure 2 describes the transfer circuit. From straightforward circuit theory, Q factors related to power dissipation by the n^2Z_0 and R elements in Fig. 2 may be obtained. Following the usual convention,[1,2] the dissipation associated with the n^2Z_0 element is characterized by an external Q factor, Q_E. The dissipation associated with the R element is characterized by a volume Q factor, Q_V. In terms of the circuit parameters and drive frequency ω, these Q factors are

$$Q_E = \omega(L + n^2L_a)/n^2Z_0, \tag{3}$$

$$Q_V = \omega(L + n^2L_a)/R. \tag{4}$$

The overall Q factor, often termed the loaded Q or Q_L, is obtained in the usual way as

$$1/Q_L = 1/Q_E + 1/Q_V. \tag{5}$$

The total circuit impedance is, from Fig. 2,

$$Z = (n^2Z_0) + R + i[\omega(L + n^2L_a) - 1/\omega C]. \tag{6}$$

With $\omega_r = 1/(L + n^2L_a)C$, we may rewrite Z as

$$Z = (n^2Z_0) + R + i\omega_r(L + n^2L_a)(\omega/\omega_r - \omega_r/\omega). \tag{7}$$

By invoking the Q_V expression in Eq. (4) the circuit impedance may be expressed in terms of an effective line impedance,

$$Z_0^{\text{eff}} = (n^2Z_0), \tag{8}$$

and an effective cavity impedance,

$$Z_C^{\text{eff}} = R + iRQ_V\Delta, \tag{9}$$

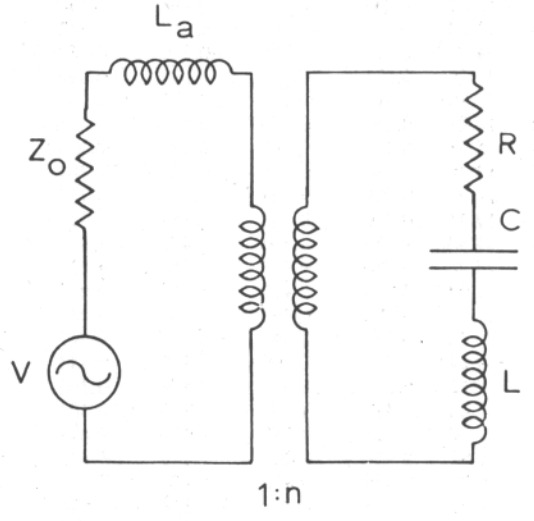

FIG. 1. Equivalent Circuit for an inductively coupled reflection cavity.

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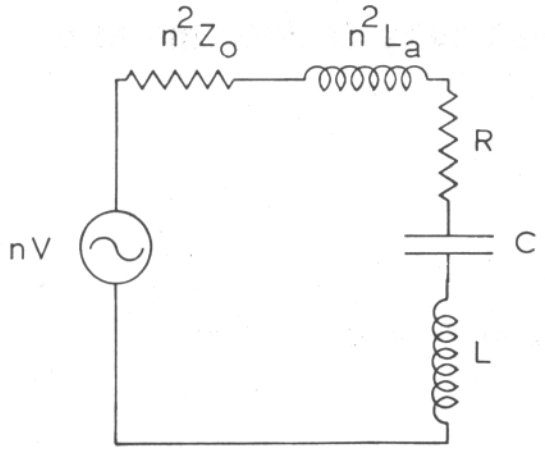

FIG. 2. Transfer circuit for the cavity circuit in Fig. 1.

where

$$\Delta = \omega/\omega_r - \omega_r/\omega. \tag{10}$$

Utilizing these impedance relations, the voltage reflection coefficient is just

$$\rho = \left| \frac{R + iRZ_V\Delta - n^2Z_0}{R + iRQ_V\Delta + n^2Z_0} \right| . \tag{11}$$

In terms of the conventional β parameter,

$$\beta = n^2Z_0/R = Q_V/Q_E, \tag{12}$$

TABLE I. Representative data for a TE101 rectangular reflection x-band cavity. nominal frequency: 8.919 GHz; nominal $\Gamma_{db}{}^0$: 16.50 dB.

f(MHz)	Γ_{db}	ρ	δ_V	δ_E	δ_L	$\Delta(\times 10^6)$
8917.50	4.77	0.577	−1.188	−1.607	−0.683	−288.6
9817.55	5.11	0.555	−1.119	−1.512	−0.643	−267.2
8917.61	5.34	0.541	−1.075	−1.453	−0.618	−253.9
8917.67	5.65	0.522	−1.019	−1.378	−0.586	−240.5
8917.73	6.08	0.497	−0.949	−1.283	−0.546	−227.2
8917.79	6.42	0.478	−0.898	−1.214	−0.516	−213.8
8917.85	6.87	0.453	−0.835	−1.129	−0.480	−200.5
9817.91	7.36	0.429	−0.773	−1.045	−0.444	−187.2
8917.97	7.87	0.404	−0.714	−0.965	−0.410	−173.8
8918.03	8.45	0.378	−0.652	−0.882	−0.375	−160.5
8918.09	8.97	0.356	−0.601	−0.813	−0.346	−147.1
8918.15	10.05	0.314	−0.507	−0.685	−0.291	−133.8
8918.27	11.01	0.282	−0.432	−0.584	−0.249	−107.1
8918.39	12.27	0.244	−0.345	−0.466	−0.198	−80.4
8918.45	13.53	0.211	−0.264	−0.357	−0.152	−67.0
8918.51	14.31	0.193	−0.215	−0.290	−0.123	−53.7
8918.57	15.20	0.174	−0.156	−0.211	−0.090	−40.3
8918.63	15.90	0.160	−0.102	−0.137	−0.058	−27.0
8918.80	16.13	0.156	0.079	0.106	0.045	12.9
8918.86	15.33	0.171	0.147	0.199	0.084	26.3
8918.92	14.70	0.184	0.190	0.257	0.109	39.6
8918.98	14.00	0.200	0.234	0.317	0.135	53.0
8919.04	13.15	0.220	0.288	0.389	0.165	66.3
8919.10	12.43	0.239	0.334	0.452	0.192	79.7
8919.16	11.70	0.260	0.383	0.518	0.220	93.0
8919.22	11.09	0.279	0.426	0.577	0.245	106.4
8919.28	10.22	0.308	0.493	0.666	0.283	119.7
8919.34	9.62	0.330	0.543	0.734	0.312	133.1
8919.40	9.00	0.355	0.599	0.809	0.344	146.4
8919.46	8.62	0.371	0.635	0.859	0.365	159.8
8919.52	8.02	0.397	0.697	0.943	0.401	173.1
8919.58	7.50	0.422	0.756	1.023	0.435	186.4
8919.64	6.97	0.448	0.822	1.112	0.473	199.8
8919.70	6.82	0.456	0.842	1.138	0.484	213.1
8919.76	6.32	0.483	0.913	1.234	0.525	226.5
8919.82	6.00	0.501	0.962	1.300	0.553	239.8

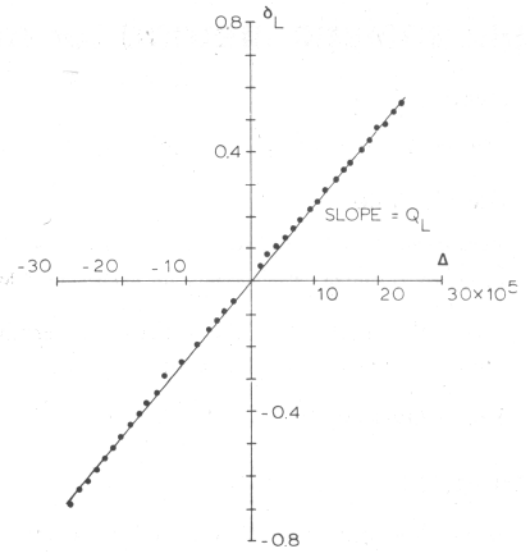

FIG. 3. Plot of δ_L vs. Δ for the test cavity.

we may write

$$\rho = \sqrt{\frac{(1-\beta)^2 + \delta_V{}^2}{(1+\beta)^2 + \delta_V{}^2}} \tag{13}$$

with

$$\delta_V = Q_V\Delta. \tag{14}$$

Solving Eq. (13) for ρ yields

$$\delta_V{}^2 = \frac{(1+\beta)^2\rho^2 - (1-\beta)^2}{1-\rho^2} . \tag{15}$$

By similar manipulations, relations between $\delta_E = Q_E\Delta$ or $\delta_L = Q_L\Delta$ and (ρ,β) may also be obtained:

$$\delta_E{}^2 = \delta_V{}^2/\beta^2, \tag{16}$$

$$\delta_L{}^2 = \delta_V{}^2/(1+\beta)^2, \tag{17}$$

with $\delta_V{}^2$ given by Eq. (15). These equations lead to a very straightforward method for the determination of Q_V, Q_E, and Q_L.

(1) Measure the power-reflection coefficient in db, Γ_{db}, over a frequency interval which straddles the cavity frequency ω_0.

(2) Determine nominal values of ω_0 and $\Gamma_{db}{}^0$ at ω_0.

(3) Calculate $\rho_0 = 10^{-(\Gamma_{db}{}^0/20)}$ and $\beta = (1-\rho_0)/(1+\rho_0)$ (undercoupled) or $\beta = (1+\rho_0)/(1-\rho_0)$ (overcoupled).

(4) Calculate $\rho = 10^{-(\Gamma_{db}/20)}$ vs. ω.

(5) Calculate δ_V, δ_E, and δ_L vs. ω from Eqs. (15–17).

(6) Do a linear least squares fit to determine an accurate ω_0, and then generate Δ vs. ω.

(7) Do a linear least square fit to the data on δ_V, δ_E, and δ_L vs. Δ. The slopes correspond precisely to Q_V, Q_E, and Q_L.

TABLE II. Fitted cavity parameters.

f = 8918.75 MHz
$Q_V = 4120 \pm 20$
$Q_E = 5570 \pm 27$
$Q_L = 2368 \pm 12$

This procedure has been used for the following example, using a conventional TE_{101} rectangular, undercoupled reflection cavity. The klystron was stabilized to ± 0.01 MHz (short term) by use of a commercial microwave stabilizer. Reflection coefficients were measured by means of a calibrated precision attenuator and a removable short at the cavity iris position. Data were obtained in steps of 0.06 MHz over a range extending from about 1 MHz below the cavity frequency f_0 to about 1 MHz above f_0. The numerical results for cavity frequency f, Γ_{db}, δ_V, δ_E, δ_L, and Δ are indicated in Table I. A plot of δ_L versus Δ is shown in Fig. 3. The cavity loaded Q, Q_L, is simply the slope of the line in Fig. 3. The cavity parameters which result from the fitting are listed in Table II.

ACKNOWLEDGMENTS

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[1] J. C. Slater, *Microwave Electronics* (Van Nostrand, New York, 1950).

[2] M. Sucher, *Handbook of Microwave Measurements*, edited by M. Sucher and J. Fox, (Wiley, New York, 1963), Vol. II, pp. 417–493.

[3] Z. Frait, Slaboproudy Obzor **20**, 144–149 (1959), in Czech.