INFLUENCE OF THE SPIN-WAVE LINEWIDTH ON THE SPIN-WAVE PROPAGATION DIRECTION FOR PARALLEL PUMPING IN SINGLE CRYSTAL YIG

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ABSTRACT

The peculiar secondary minimum in the parallel pump butterfly curve for [110] magnetized single crystal YIG, originally discovered by Sethares and coworkers, has been experimentally and theoretically investigated. Data were obtained for an in-plane [110] magnetized ($1\bar{1}0$) YIG slab at 9.197 GHz and room temperature. The data were analyzed by the parallel pump theory of Haubenreisser, which includes the effect of crystalline anisotropy, with the wave-vector dependence of the spin-wave linewidth taken into account. Except for a narrow 40 Oe field interval about 50 Oe below the usual butterfly curve minimum, the instability threshold is lowest for spin waves propagating perpendicular to the slab plane. Over this narrow interval, however, the k-dependence of the spin-wave linewidth combines with the anisotropy influence on the spin-wave manifold to flip the unstable spin waves to be in-plane. This causes the secondary butterfly curve minimum.

I. INTRODUCTION

Sethares et al.[1] have reported an anomalous secondary minimum in the parallel pump butterfly curve for single crystal YIG. The effect was largest for samples magnetized along [110]. The data implied an abrupt 90° change in the azimuthal propagation direction for the unstable spin waves over a narrow field interval around the secondary minimum. No theoretical justification for this change was found. The theory of parallel pumping in cubic single crystal ferrites has been considered by Haubenreisser and coworkers[2,3]. Reference 3 further examines but does not explain the secondary minimum effect.

In the present study, the parallel pump threshold h_{crit} was measured as a function of static field H for single crystal ($1\bar{1}0$) YIG plates magnetized along [110],[4] using the cw technique developed by Jantz.[5] A distinct secondary minimum similar to that reported in ref. 1 was found. A detailed consideration of the Haubenreisser theory and the shape of the spin-wave manifold for single crystals, combined with the dependence of the spin-wave linewidth ΔH_k on spin-wave wavevector k, led to a rather simple explanation of the secondary minimum. The linear k-dependence of ΔH_k promotes the pumping of modes parallel to [001], while the parallel pump coupling favors spin waves along [$1\bar{1}0$]. For most of the field interval over which parallel pumping is observed, the coupling effect dominates. For a small field interval below the usual minimum, however, it is found that the influence of ΔH_k is strong enough to flip the unstable modes by 90°, causing the secondary minimum.

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II. EXPERIMENTAL RESULTS

Parallel pump data were obtained on a ($1\bar{1}0$) plane single crystal YIG slab. A Varian V-4500 EPR spectrometer at 9.197 GHz and room temperature was used. Figure 1 illustrates the sample geometry and spin-wave propagation parameters. Spin waves of wave number k, with polar and azimuthal angles of Θ_k and ϕ_k, respectively, are indicated. The static field H, magnetization M_o, and microwave field h are all along [110], in the plane of the slab. The butterfly curve data were obtained by sweeping the static field with the microwave field held constant, and noting the instability threshold points as discussed in ref. 5.

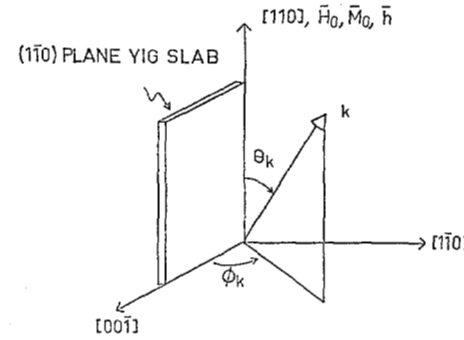

Fig. 1. Sample and spin wave geometry.

The measured values of the parallel pump threshold h_{crit} versus static field are given by the solid points in Fig. 2. The results for $H > H_o \approx 950$ Oe correspond to the usual butterfly curve for parallel pumping with the minimum threshold at $H = H_c \approx 995$ Oe. As noted in refs. 1 and 3, this corresponds precisely to the predicted theoretical minimum for spin waves with $\Theta_k=90°$ and $\phi_k=90°$ in the k=0 limit. For $H < H_o$, however, the threshold decreases to a secondary minimum at $H=H_c'$, rather than continue to increase as the theory predicts. It was noted that the minimum h_{crit} position at H_c' corresponds precisely to the static field for $\phi_k=0°$ spin waves in the k=0 limit.[1] Such modes should have a higher threshold than $\phi_k=90°$ spin waves and should therefore not be observed in the parallel pump experiment. The secondary minimum presents a further problem. It implies that only $\phi_k=0°$ or $\phi_k=90°$ spin waves are pumped and that intermediate values are avoided. Otherwise, a continuous decrease in threshold would occur in going from H_c' to H_c!

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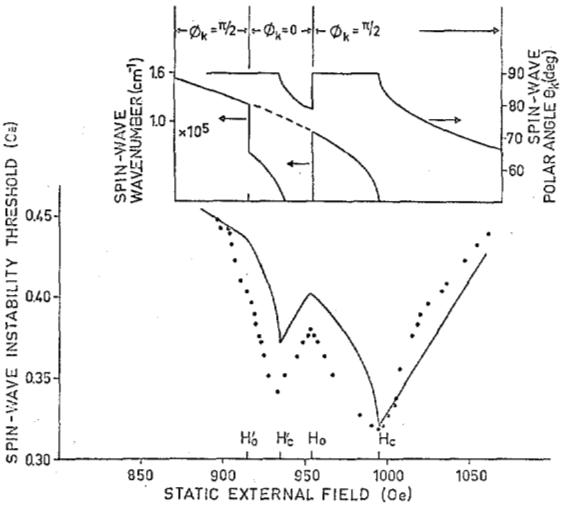

Fig. 2. Experimental and theoretical parallel pump butterfly curves for single crystal YIG (110) slab magnetized along [110], at 9.197 GHz and room temperature.

III. THEORETICAL RESULTS

A detailed analysis of the data, based on the parallel pump theory for single crystals which includes the effect of anisotropy on the spin-wave linewidth and the k-dependence of the spin-wave linewidth, appears to explain the secondary minimum effect.

First consider the theoretical threshold h_c for spin waves of frequency $\omega_k = \omega/2$, where ω is the pump frequency in rad/sec, with a direction defined by θ_k and ϕ_k in Fig. 1.

$$h_c = (\omega/\omega_m)(\Delta H_k/\sin^2\theta_k)\cdot \quad (1)$$
$$\cdot\left[1 + (3H_k/4\pi M_o\sin^2\theta_k)^2 - (6H_k/4\pi M_o\sin^2\theta_k)\cos(2\phi_k)\right]^{-1/2}$$

Equation (1) applies specifically to [110] magnetized YIG with an anisotropy field $H_k=-K_1/M_o$, where K_1 is the negative first order crystalline anisotropy constant. For YIG, H_k is about 43 Oe. In Eq.(1), ω_m is equal to $\gamma 4\pi M_o$, where γ is the gyromagnetic ratio, 1.76×10^7 Oe^{-1}Sec^{-1}. The $(\omega/\omega_m)(\Delta H_k/\sin^2\theta_k)$ term is the same as for isotropic materials. The square bracket term (SBT) represents the effect of anisotropy on the parallel pump coupling. The observed threshold h_{crit} corresponds to the minimum of h_c when all available spin-waves states at $\omega_k=\omega/2$ are considered. The SBT in Eq.(1) favors the instability of spin waves with $\phi_k=90^o$. The explicit θ_k terms in Eq.(1) combine to favor modes with $\theta_k=90^o$, just as for isotropic materials.

The available spin-wave modes at $\omega_k=\omega/2$ are limited by the spin-wave dispersion constraint. When first order anisotropy is included, the dispersion is given by[2]

$$(\omega_k/\gamma)^2 = (H + Dk^2 + 4\pi M_o\sin^2\theta_k - H_k)\cdot$$
$$\cdot(H + Dk^2 + 2H_k) - 3H_k 4\pi M_o\sin^2\theta_k\sin^2\phi_k, \quad (2)$$

where $D=4.5 \times 10^{-9}$ Oe cm^2 is the exchange parameter for YIG. Equation (2) shows that for [110] magnetized YIG, ω_k depends on ϕ_k as well as on θ_k. The dispersion is shown schematically in Fig. 3, for a static field somewhat below H_c'. The ϕ_k dependence in Eq.(2) changes the $\theta_k=90^o$ branch line of the manifold into a branch surface. A $\omega_k=\omega/2$ cut through that surface is indicated in the figure by the line P-Q. Spin waves with $\theta_k=90^o$ and $\omega_k=\omega/2$ are constrained so that k is an increasing function of ϕ_k. Thus, spin waves with $\phi_k=90^o$, which from Eq.(1) would have the lowest threshold, must also have the largest k.

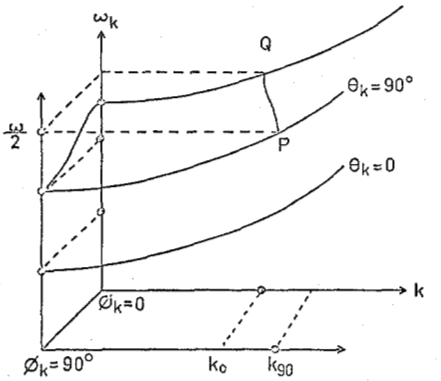

Fig. 3. Spin-wave dispersion for single crystal YIG magnetized along [110]. Curve P-Q indicates the cut of $\omega_k=\omega/2$ through the $\theta_k=90^o$ branch surface.

The key to the origin of the secondary minimum lies in the realization that the spin-wave linewidth ΔH_k, which appears in the numerator of Eq.(1), has a linear k-dependence.[6] Because of the manifold constraint depicted in Fig. 3 and discussed above, this means that spin waves with the lowest ΔH_k will also have $\phi=0^o$. We therefore have two opposing tendencies. The coupling favors $\phi_k=90^o$ spin waves, while the spin-wave linewidth k-dependence combines with the dispersion constraint to favor $\phi_k=0^o$ spin waves. In the light of these considerations, the data suggest that the ΔH_k influence takes over in the field interval $H_o' < H < H_o$ to flip the instability to $\phi=0^o$ spin waves and cause the secondary minimum.

In order to quantitatively check this hypothesis, it is necessary to select some reasonable trial function for $\Delta H_k(k,\theta_k,\phi_k)$. A function of the form

$$\Delta H_k = A + Bk + C\sin^2 2\theta_k + E\sin^2 2\phi_k \quad (3)$$

was chosen, where A, B, C, and E are adjustable parameters. Only the A + Bk term has a solid theoretical justification from first principles. It is related to Kasuya-LeCraw and three magnon relaxation processes.[7] The $\sin^2 2\theta_k$ dependence was found to provide the best fit to data above $H=H_o$, among various simple trigonometric functions of θ_k. The exact form of this term is not of vital importance as far as the secondary minimum is concerned. The $\sin^2 2\phi_k$ term is included to account for the empirical preference of ϕ_k for 0^o or 90^o, as noted in ref. 1. The numer-

ical values for A, B, and C were obtained by fitting the data to the theory at three static field points, one at the high field limit of the data, one at the low field limit, and one at $H=H_c$. It is to be noted that none of these points pertain to the secondary dip. The values for A, B, and C were determined to be 0.018 Oe, 5.64×10^{-7} Oe cm^2, and 0.060 Oe, respectively. The E parameter was set at an arbitrary large value of 5 Oe simply to provide the above noted constraint on ϕ_k. This last term will be considered in more detail shortly.

The solid curve in Fig. 2 shows the theoretical butterfly curve that came out of the analysis. The secondary minimum is clearly in evidence. The insert in the figure shows the k, θ_k, and ϕ_k dependences on static field as well. For $H < H_o'$, the reduction in ΔH_k due to the decrease in k at $\phi_k=0^o$ is not sufficient to overcome the coupling preference for $\phi_k=90^o$. As H exceeds H_o', however, the ΔH_k effect takes over and flips the unstable mode to $\phi_k=0^o$. This is intuitively reasonable, since the reduction in k which is needed to maintain ω_k at $\omega/2$ as ϕ_k changes from 90^o to 0^o is approximately

$$\Delta k \approx 3H_k \cdot 4\pi M_o \left[\partial (\omega_k/\gamma)^2/\partial k\right]^{-1} \propto k^{-1} \quad (4)$$

Thus, the relative effect of the ϕ_k flip on ΔH_k will increase as k decreases, or as H increases. As H exceeds H_c', the $\phi_k=0^o$ preference, together with the dispersion constraint, forces θ_k to decrease from 90^o and h_{crit} therefore increases. At the point $H=H_o$, this continuing increase in h_{crit} makes it more favorable that ϕ_k and θ_k flip back to 90^o, in spite of the necessary increase in k and ΔH_k. It is seen that the secondary minimum region is really the small observable portion of a second butterfly curve for $\phi_k=0^o$, just as the larger field region corresponds to the $\phi_k=90^o$ butterfly curve. It is to be emphasized that none of the fitted parameters in ΔH_k pertain explicitly to the secondary dip. Yet, the dip itself and the break points which come out of the analysis are in agreement with the data. The remaining difference between the theory and the data lies only in the prominence of the additional minimum. This discrepancy may well be due to the sample shape. Since $\phi_k=0^o$ magnons propagate in the plane of the platelet, this may give an additional preference for $\phi_k=0^o$.

The analysis does not explain why $\phi_k=0^o$ or $\phi_k=90^o$ magnons only are observed, and that intermediate values of ϕ_k are avoided. The following mechanism is proposed. Consider a magnon at $\omega_k=\omega/2$ with a general ϕ_k. This magnon is pumped by the microwave field and is also coupled to degenerate near-by magnon states by two-magnon interactions. If the pumped mode can transfer energy to these degenerate states, the ΔH_k and the threshold of the pumped mode will be increased. In effect, the degenerate states act as energy sinks to increase the h_c of the pumped mode. This is possible only if the degenerate states are less efficiently excited by the microwave field than is the pumped mode under consideration. This is generally the case, since the coupling term in Eq.(1) is a function of ϕ_k. From Eq.(1), however, it is seen that both $\partial h_c/\partial \theta_k$ and $\partial h_c/\partial \phi_k$ vanish for the special cases of $\theta_k=90$ and $\phi_k=0,90^o$. Under these conditions, the near-by states are comparably pumped and cannot act as energy sinks to increase the ΔH_k of these special magnons. Such a process may be the theoretical basis for the observation of only $\phi_k=0^o$ or $\phi_k=90^o$ magnons in parallel pumping.

IV. CONCLUSION

The present results provide a new perspective on the phenomenon of spin-wave instability in general and parallel pumping in particular. It is seen that parallel pumping can involve more than simply "dialing up" a given magnon. As one traverses the butterfly curve, the selected mode may abruptly flip from one k-value to another. It has recently become possible to use light scattering techniques to detect thermal magnons in YIG directly.[8] It should be possible to use such techniques to actually observe the directional flip for the pumped magnons discussed above. In terms of theory, there is no satisfactory quantitative explanation of the mechanism which leads to the selection of $\phi_k=0^o$ or $\phi_k=90^o$ modes only.

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