CLASSICAL THEORY OF SPIN-WAVE DISPERSION FOR FERROMAGNETIC METALS*)

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The properties of the four spin-wave dispersion branches for a planar ferromagnetic metal with TEM microwave excitation at normal incidence have been examined to derive physical insight into (a) their dependence on material parameters and orientation, (b) the interaction between branches, and (c) the polarization character associated with the related ecxitations. The Larmor electromagnetic and spin-wave branches are found to exhibit a peculiar crossover effect at a critical damping value ($\alpha \sim 0{\cdot}014$ for permalloy). The polarizations vary from circular for the static field normal to the surface to elliptical for an in-plane static field. For the in-plane case, the major polarization axes are parallel and perpendicular to the surface for the Larmor and antilarmor branches, respectively. The well known Ament-Rado and Vittoria formulations have also been modified to include the effect of exchange in the damping term of the Landau-Liftshitz equation. The secular determinant is simplified considerably and the form of the dispersion equations is changed significantly. The only large effect on the branches themselves is to increase the attenuating component of the propagation parameter k for large $|k|$.

INTRODUCTION

The classical theory of ferromagnetic resonance for isotropic metals of AMENT and RADO [1], SOOHOO [2], and VITTORIA and co-workers [3], is well known and widely used to interpret FMR phenomena in single crystal whiskers and platelets, as well as thin films. The dispersion behavior of the normal spin-wave modes, to which the FMR response is ultimately attributed, has not been studied in any systematic manner. SOOHOO [2] has presented a brief description of the dispersion properties for the case of large damping, primarily for the spin-wave branches. VITTORIA and co-workers [3] have discussed the dispersion curve behavior, in the high damping limit, in some detail and described the basic shapes of the spin-wave and electromagnetic branches in this limit (item 1 in Ref. [3]).

The present study was concerned with a study of the four characteristic dispersion branches, based primarily on the theoretical formulations cited above, with a focus on (1) the evolution of and interaction between the branches as a function of the material parameters of the metallic ferromagnet, and (2) the nature of the normal mode excitations associated with those branches. Consistent with these objectives, the effect of damping and conductivity on the dispersion has been examined, and the polarization character of the four dispersion branches has been derived. The results provide new insight concerning the normal mode collective excitations in ferromagnetic metals.

THEORY

Following the method of VITTORIA et al. [3], one may write the equation of motion,

$$\mathrm{d}\boldsymbol{M}/\mathrm{d}t = \gamma \boldsymbol{M} \times [\boldsymbol{H}_0 + \boldsymbol{h} + \boldsymbol{h}_{ex} + \boldsymbol{h}_{\lambda}], \tag{1}$$

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in terms of a sum of field quantities which characterize the different contributions to the dynamic response. $\boldsymbol{H}_0$ is the static internal field, $\boldsymbol{h}$ is the internal microwave field, $\boldsymbol{h}_{ex}$ is an effective microwave field due to the exchange coupling between non-parallel spins, and $\boldsymbol{h}_\lambda$ is an effective damping field derived using the Landau-Liftshitz form. The quantity γ denotes the gyromagnetic ratio ($\sim -1{\cdot}76 \times 10^7$ sec^{-1} Oe^{-1}, negative for electrons), and $\boldsymbol{M}$ is the local vector magnetization. For small excitations, we may write,

$$\boldsymbol{M} = \boldsymbol{M}_0 + \boldsymbol{m} . \tag{2}$$

$\boldsymbol{M}_0$ denotes the static component with magnitude $|\boldsymbol{M}_0| = M_0$, where $4\pi M_0$ is the saturation induction, and $\boldsymbol{m}$ is the time dependent component. With $|\boldsymbol{m}| \ll M_0$ for small excitations, $\boldsymbol{m}$ is perpendicular to $\boldsymbol{M}_0$ to first order.

The effective field terms in Eq. (1), $\boldsymbol{h}_\lambda$ and $\boldsymbol{h}_{ex}$, are specified by,

$$\boldsymbol{h}_\lambda = -(\lambda/\gamma M_0^2)(\boldsymbol{M}_0 + \boldsymbol{m}) \times (\boldsymbol{H}_0 + \boldsymbol{h} + \boldsymbol{h}_{ex}) , \tag{3a}$$

$$\boldsymbol{h}_{ex} = (2Ak^2/M_0^2)\,\boldsymbol{m} , \tag{3b}$$

where A is the usual exchange stiffness constant ($\sim 1{\cdot}0 \times 10^{-6}$ erg/cm), and λ denotes the so-called Landau-Liftshitz relaxation frequency (in rad/sec). The spatial dependence of the microwave quantities, $\boldsymbol{h}$ and $\boldsymbol{m}$, is characterized by the complex propagation constant k defined by,

$$\boldsymbol{m}(y) \propto \boldsymbol{h}(y) \propto \mathrm{e}^{ky + i\omega t} \tag{4}$$

where the y-coordinate is perpendicular to the assumed planar surface of the ferromagnetic metal. This spatial dependence is such that the real part of the propagation constant k corresponds to an exponential growth or decay in amplitude, while the imaginary part corresponds to plane-wave propagation parallel or antiparallel to the y-axis.

It is to be noted in passing, that Eq. (3a) for the Landau-Lifshitz damping field $\boldsymbol{h}_\lambda$ is modified from the Vittoria approach insofar as the exchange contribution is explicitly included. Wilts and Ramer [4] have pointed out the importance of this contribution in fitting the full metal FMR theory to data for the linewidths of high order spin-wave modes. Soohoo had avoided this problem by using the Gilbert damping form in his theory [2]. The term has little effect on the overall shape of the dispersion curves, but is important for the linewidths of large wave-vector spin waves. The inclusion of the term, however, does provide for a somewhat simplified secular determinant for the dispersion solutions and hence, leads to somewhat modified dispersion equations as compared to Ref. [3].

The basic approach is to first evaluate the equation of motion in terms of $\boldsymbol{M}_0$, $\boldsymbol{H}_0$, and the other "intrinsic" quantities to obtain a set of first order equations relating $\boldsymbol{m}$ and $\boldsymbol{h}$. The electromagnetic conditions on $\boldsymbol{m}$ and $\boldsymbol{h}$ from Maxwell's equations are

then used to eliminate $\boldsymbol{h}$ from the equation of motion to obtain a set of homogeneous equations in $\boldsymbol{m}$. The condition for a nontrivial solution is that the determinant of coefficients vanish, which leads to a set of dispersion relations for the normal modes of the system. The actual mechanics of the analysis are identical to Ref. [3]. With the extended $\boldsymbol{h}_\lambda$ of Eq. (3a), one obtains the following vector equation in $\boldsymbol{m}$ and $\boldsymbol{h}$,

$$[i\omega/\gamma + (\lambda/\gamma M_0)]\,(H_0 - 2Ak^2/M_0)\,\boldsymbol{m} = \tag{5}$$

$$= -(\boldsymbol{M}_0/M_0) \times \boldsymbol{m}(H_0 - 2Ak^2/M_0) + \boldsymbol{M}_0 \times \boldsymbol{h} + (\lambda/\gamma)\,(\boldsymbol{h} - (\boldsymbol{U} \,.\, \boldsymbol{h})\,\boldsymbol{U}]\,,$$

where $\boldsymbol{U}$ is a unit vector in the direction of $\boldsymbol{M}_0$. This equation differs from Ref. [3], in that a single effective static field $(H_0 - 2Ak^2/M_0)$ appears on both sides.

The usual conditions on $\boldsymbol{m}$ and $\boldsymbol{h}$ from Maxwell's equations are used to eliminate $\boldsymbol{h}$ in the equation of motion [1, 2, 3] and obtain a three by three secular determinant, $\boldsymbol{A}$, which must vanish if nontrivial $\boldsymbol{m}(y)$ solutions are to exist. The components A_{ij} are given by:

$$A_{xx} = -Lb(1 - \alpha_1^2) - i\omega/\gamma \tag{6a}$$

$$A_{xy} = c(\alpha_3 + L\alpha_1\alpha_2) \tag{6b}$$

$$A_{xz} = b(-\alpha_2 + L\alpha_1\alpha_3) \tag{6c}$$

$$A_{yx} = b(-\alpha_3 + L\alpha_1\alpha_2) \tag{6d}$$

$$A_{yy} = -Lc(1 - \alpha_2^2) - i\omega/\gamma \tag{6e}$$

$$A_{yz} = b(\alpha_1 + L\alpha_2\alpha_3) \tag{6f}$$

$$A_{zx} = b(\alpha_2 + L\alpha_3\alpha_1) \tag{6g}$$

$$A_{zy} = c(-\alpha_1 + L\alpha_2\alpha_3) \tag{6h}$$

$$A_{zz} = -Lb(1 - \alpha_3^2) - i\omega/\gamma \tag{6i}$$

where $(\alpha_1, \alpha_2, \alpha_3)$ are the direction cosines of $\boldsymbol{M}_0$ and $\boldsymbol{H}_0$ relative to the (x, y, z) axes, L is a reduced damping $(L = \lambda/\gamma M_0)$ in the Ament-Rado notation, and the field parameters b and c are given by:

$$b = H_0 - 2Ak^2/M_0 + M_0/Q\,, \tag{7a}$$

$$c = H_0 - 2Ak^2/M_0 + 4\pi M_0\,, \tag{7b}$$

with $Q = (1 + ik^2\delta_0^2/2)/4\pi$. δ_0 is the classical electromagnetic skin depth $c/\sqrt{(2\pi\omega\sigma)}$ for a metal of conductivity σ.

For any particular orientation of the magnetization relative to the film plane, the secular determinant can be solved for the four dispersion curve branches, in terms of ω versus complex k. Results for the perpendicular ($\alpha_1 = \alpha_3 = 0$, $\alpha_2 = 1$) and parallel ($\alpha_1 = 1$, $\alpha_2 = \alpha_3 = 0$) orientations are particularly useful. In the perpendicular case, two quadratic equations are obtained. For parallel, one obtains the usual skin-depth equation in k^2 and a cubic equation in k^2.

It is convenient to express the field, frequency, damping, exchange, and propagation constant in terms of reduced variables, following the convention introduced by AMENT and RADO in Ref. [1].

Reduced field:	$\eta = H_0/4\pi M_0$
Reduced frequency:	$\Omega = \omega/\gamma\, 4\pi M_0$
Reduced damping:	$L = \lambda/\gamma M_0$
Reduced exchange:	$\varepsilon^2 = A/(2\pi M_0^2 \delta_0^2)$
Reduced propagation constant:	$K = k\varepsilon\delta_0 = k \,.\, \sqrt{(A/2\pi M_0^2)}$

For common transition metal ferromagnets, γ is a negative quantity. Thus, with ω and λ as positive quantities, the corresponding reduced variables Ω and L are negative. Note also that the Ament-Rado reduced damping L is the negative of the commonly used Landau-Lifshitz damping parameter α.

In terms of these reduced variables, the dispersion equations which result after the algebraic manipulations are as follows:

Perpendicular

$$K^4 + K^2[\Omega/(\mathrm{i}L \pm 1) - \eta - 2\mathrm{i}\varepsilon^2] - 2\mathrm{i}\varepsilon^2[\Omega/(\mathrm{i}L \pm 1) - (\eta + 1)] = 0\,. \tag{8}$$

Parallel

$$2\varepsilon^2 + \mathrm{i}K^2 = 0\,, \tag{9a}$$

$$K^6 - C_1K^4 + C_2K^2 - C_3 = 0\,, \tag{9b}$$

with

$$C_1 = 2\mathrm{i}\varepsilon^2 + 2\eta + 1 + 2\mathrm{i}L\Omega/(1 + L^2)\,, \tag{10a}$$

$$C_2 = (\eta + 4\mathrm{i}\varepsilon^2)(\eta + 1) - [\Omega/(1 + L^2)]\,[\Omega + L\{4\varepsilon^2 - \mathrm{i}(2\eta + 1)\}]\,, \tag{10b}$$

$$C_3 = 2\mathrm{i}\varepsilon^2[(\eta + 1)^2 - \{\Omega/(1 + L^2)\}\,\{\Omega - 2\mathrm{i}L(\eta + 1)\}]\,. \tag{10c}$$

These expressions differ from the algebraic expressions of AKHIEZER [5] for perpendicular and AMENT and RADO [1] for parallel, because of the additional term in $\boldsymbol{h}_\lambda$.

DISPERSION CHARACTER

The above relations can be used to generate actual dispersion curves for the normal mode magnetic excitations in metallic ferromagnets. The general character of the dispersion can be easily extracted by looking at the limiting equations for zero damping and zero conductivity. In this limit, one obtains,

$$K^2(K^2 + \Omega - \eta) = 0\,, \tag{11a}$$

$$K^2(K^2 - \Omega - \eta) = 0\,, \tag{11b}$$

for perpendicular, and

$$K^2 = 0\,, \tag{12a}$$

$$K^2[K^4 - (2\eta + 1)K^2 + \eta(\eta + 1) - \Omega^2] = 0 \tag{12b}$$

for parallel.

In each case, there are four (K^2) roots. For perpendicular, each equation yields one $(K^2) = 0$ root and one other root, $(K^2) = \eta \pm \Omega$. For parallel, we have a single $(K^2) = 0$ root from the first equation, and from the second there is one $(K^2) = 0$ root and two roots of the factor in square bracket. When conductivity is included in the analysis, the two $(K^2) = 0$ roots for each case will evolve as electromagnetic dispersion branches. For the insulator limit, these branches have simply folded into the $|\Omega|$ axis. The other two roots correspond to the spin-wave branches in the lossless insulator limit.

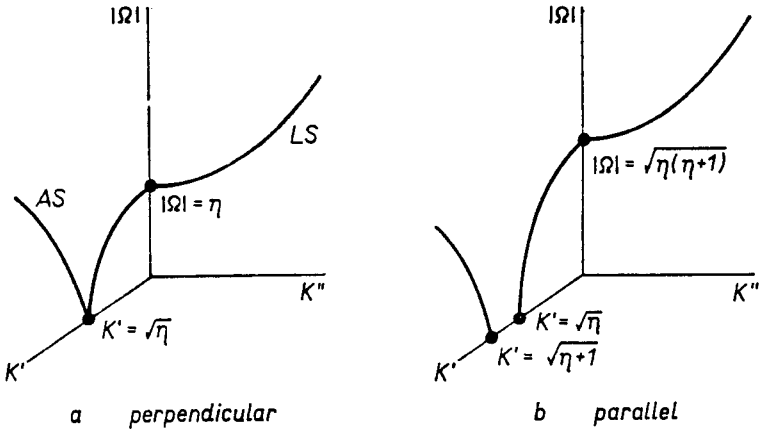

Fig. 1. Schematic $|\Omega|$ vs. K dispersion curves for the two spin-wave branches in the (a) perpendicular and (b) parallel configurations.

These latter two roots can be used to generate easily visualized dispersion curves for the lossless insulator limit, with the curves displayed as $|\Omega|$ versus complex $K = K' + iK''$, where K' and K'' are real numbers. Some qualitative results for perpendicular and parallel are shown schematically in Fig. 1. In both cases, the allowed propagation constants for small $|\Omega|$ are real, corresponding to exponentially damped or growing excitations (as a function of y). For perpendicular, the two branches both start out with $K' = (\eta)^{1/2}$ at $|\Omega| = 0$. One of the branches moves out in K' as $|\Omega|$ increases. The other branch exhibits a decreasing K' with $|\Omega|$. At $|\Omega| = \eta$, the propaga-

tion changes from purely real to purely imaginary and $|\Omega|$ continues to increase as $|\Omega| - \eta \propto (K'')^2$. This latter region, with imaginary K, corresponds to a propagating, plane wave type excitation. For parallel, the behavior is similar except that the two branches are split at $|\Omega| = 0$, and the conversion from real K to imaginary K occurs at $|\Omega| = \sqrt{[\eta(\eta + 1)]}$. It will prove useful to denote this field (and orientation) dependent frequency point at which K converts from real to imaginary as "crossover". For reasons to be apparent later, we may also denote the branch which exhibits crossover and the branch which moves out in K' with increasing $|\Omega|$ as the

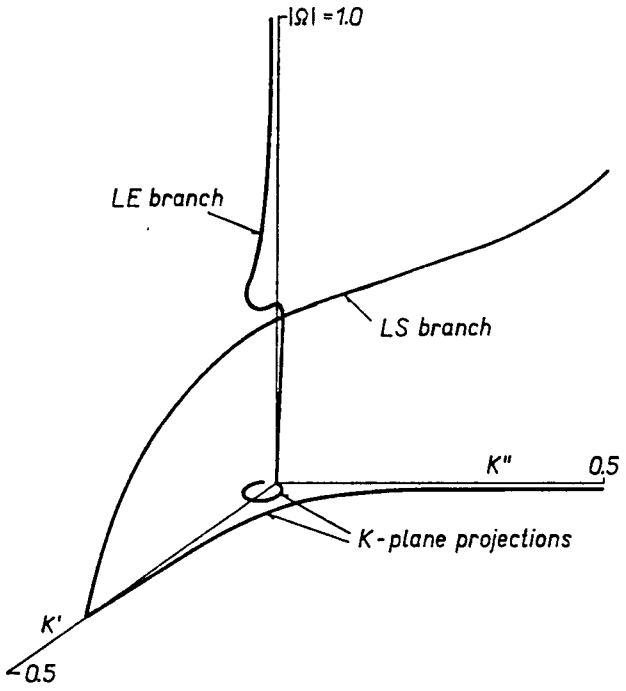

Fig. 2. Behavior of the Larmor spin-wave (LS) and Larmor electromagnetic (LE) branches for a non-zero conductivity and large damping. The curves shown are also presented in K-plane projection. Permalloy material parameters and $\alpha = 0{\cdot}030$ were used to generate the curves for the parallel configuration under bias conditions equivalent to insulator FMR at 10 GHz ($|\Omega| = 0{\cdot}337$) for $H_0 = 1135$ G($\eta = 0{\cdot}113$).

Larmor spin-wave branch (LS) and the antilarmor spin-wave branch (AS), respectively. Expressions similar to those for parallel and perpendicular can be evaluated for other magnetization directions. For a magnetization direction in the $x-y$ plane, specified by direction cosines α_1 or $\alpha_2(\alpha_1^2 + \alpha_2^2 = 1, \alpha_3 = 0)$, the spin-wave branches are given by,

$$K^4 - K^2(2\eta + 1 - \alpha_2^2) + \eta(\eta + 1 - \alpha_2^2) - \Omega^2 = 0\,, \tag{13}$$

which reduces to the perpendicular and parallel expressions for $\alpha_2 = 1$ and $\alpha_2 = 0$, respectively. As the orientation swings from one limit to the other with internal field H_0 (and η) held constant, the branches sweep out a general spin-wave manifold.

The effect of nonzero conductivity and relatively large damping ($\alpha = 0{\cdot}03$) on dispersion branches is shown in Fig. 2, where the dispersion equations for the parallel case have been solved by computer for field conditions equivalent to insulator FMR at 10 GHz and material parameters representative of permalloy, as summarized in Table I. In this perspective plot, curve projections on the (K', K'') plane are also shown. The scale for the K axes is too expanded to show the AS antilarmor spin-

wave branch. The Larmor spin-wave (LS) branch assumes a small positive K'' for $|\Omega|$ below crossover, and a small K' for $|\Omega|$ above crossover, with the net effect that an actual crossover at $K = 0$ is avoided. The more interesting effects of conductivity and loss show up in the electromagnetic branches. On the scale of Fig. 2, the emergence of a non-trivial electromagnetic branch is clearly evident. With $|\Omega|$ increasing from zero, this branch emerges from $K = 0$ with roughly equal real and imaginary parts.

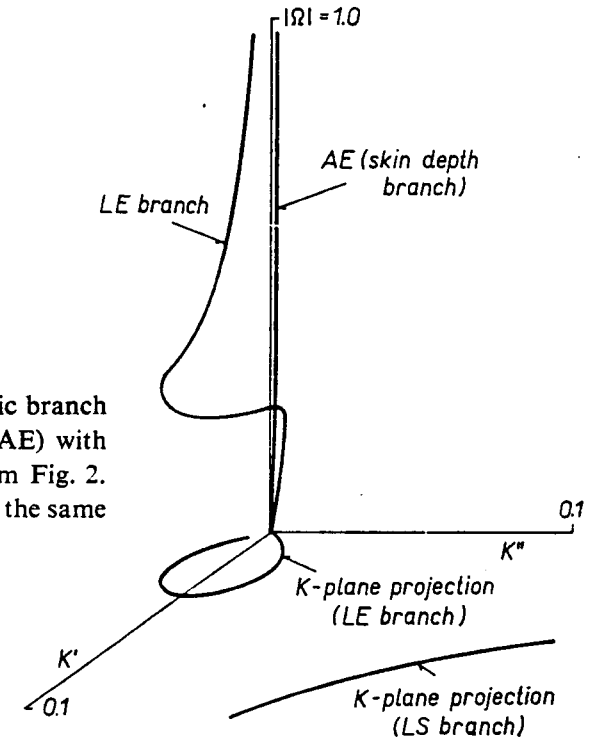

Fig. 3. Behavior of the Larmor electromagnetic branch (LE) and antilarmor electromagnetic branch (AE) with the K-scales expanded by a factor of five from Fig. 2. The parameters used to generate the curves are the same as in Fig. 2.

Table I

Material parameters and reduced quantities for a representative ferromagnetic transition metal alloy—permalloy.

Magnetization	$M_0 = 800$ G
Resistivity	$\varrho = 15\ \mu\Omega - \text{cm}$
Conductivity	$\sigma = 6 \times 10^{16}\ \text{sec}^{-1}$ (cgs)
Exchange constant	$A = 1{\cdot}0 \times 10^{-6}\ \text{erg/cm}$
Gyromagnetic ratio	$\gamma = -1{\cdot}84 \times 10^{7}\ \text{Oe}^{-1}\ \text{sec}^{-1}$

As $|\Omega|$ continues to increase, the curve bends around, K'' changes sign, and K' goes through a maximum at about the point in $|\Omega|$ where the LS branch has its closest approach to the $|\Omega|$ axis. The branch projection on the K plane describes a simple loop. This particular branch has properties of the mode excited in ferromagnetic resonance. At about the field required for resonance from the Kittel condition, $|\Omega| = \sqrt{[\eta(\eta + 1)]}$, the attenuation constant K' goes through maximum (correspond-

ing to a resonance absorption peak), and the phase of the propagation constant K'' changes sign (corresponding to the zero crossing of the dispersive part of the resonance). Let this branch be denoted as Larmor-electromagnetic (LE).

The same curves are shown on an even more explanded scale in Fig. 3. On this scale the short arc represents that part of the LS branch at the point of closest approach to the $|\Omega|$ axis. Only its K-plane projection is shown. The LE branch is shown in more

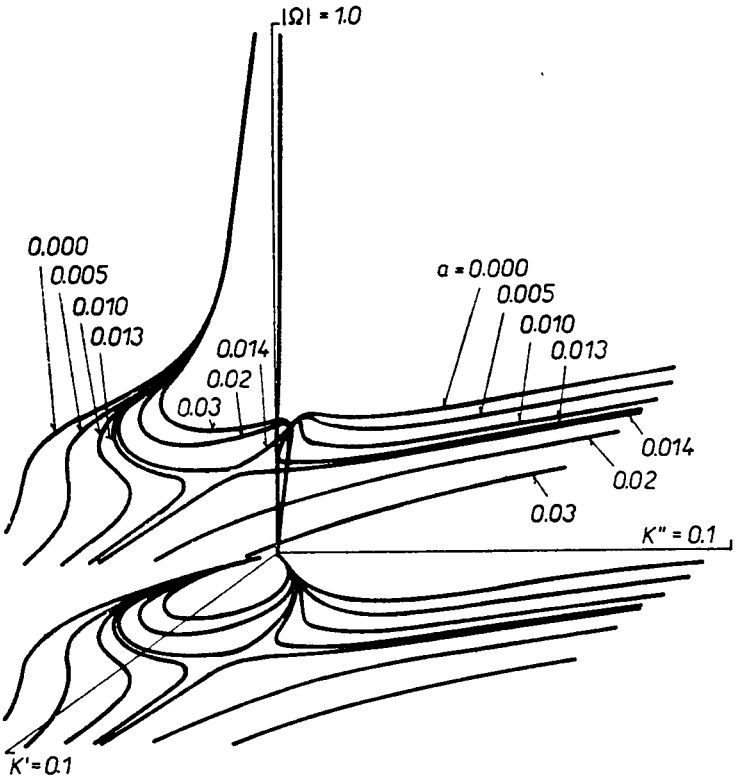

Fig. 4. Behavior of Larmor electromagnetic branch (LE) and Larmor spin-wave branch (LS) with decreasing damping from $\alpha = 0{\cdot}030$ to $\alpha = 0{\cdot}000$. The other parameters are the same as for Figs. 2 and 3.

detail. At the highest frequency point shown, $|\Omega| = 1$, the curve is moving back toward the positive K'' region. The point where K'' actually goes positive, which occurs for $|\Omega| > 1$, corresponds to the so-called antiresonance condition [6]. On this scale, a fourth branch can also be seen which simply moves out in K and up in $|\Omega|$ with $K' = K''$. This branch, which will be called the antilarmor electromagnetic branch (AE), is simply the solution to the skin depth equation, $Q = 0$, and represents the classical skin depth solution for nonmagnetic conductors. The mode associated with this branch is, in fact, uncoupled from $\boldsymbol{M}$ and gives no microwave magnetization response.

Thus, we have four dispersion branches, each with a distinct character. (1) The AS branch has a K which is almost purely real, corresponding to a highly damped excitation. (2) The LS branch has an inherently propagating spin-wave character at high $|\Omega|$, and a damped character at low $|\Omega|$. (3) The LE branch had $|K|$ near zero for all $|\Omega|$ (smaller that $|K|$ for the LS and AS branches, in any case), and exhibits the properties of ferromagnetic resonance and antiresonance. (4) The AE branch is identical with the classical dispersion for the skin-depth mode in nonmagnetic metals.

The preceding description of the branch behaviour applies for relatively large values of the damping. As indicated, the computer analysis was done for $\alpha = 0{\cdot}030$. For smaller values of the damping, the shapes of the branches in the vicinity of ferromagnetic resonance are changed considerably. The basic effect is shown in Fig. 4. Curves similar to those in Fig. 3 are shown for values of the Landau-Liftshitz damping from $\alpha = 0{\cdot}03$ down to zero. Note that only two branches are involved, the LE and the LS. The AE and AS branches are not affected significantly. For large damping, the LE and LS branches are completely separate, and behave as shown previously in Fig. 2 and Fig. 3. Figure 4 shows that as the damping is decreased, the LS branch moves in toward the $|\Omega|$ axis and the loop of the LE branch (as indicated by its projection on the (K', K'') plane) expands. At some critical value of the damping, between 0·013 and 0·014 for permalloy and $\eta = 0{\cdot}113$, the two branches touch. Both the perspective plots and the K plane projection plots show this effect. As the damping is further decreased, continuous branches evolve as "LE to LS" and "LS to LE" branches. In the vicinity of the ferromagnetic resonance biassing point, each branch is really a mixture, with both LS and LE character. Thus, it is seen that the character of the Larmor dispersion branches is quite different in the limits of low and high damping. For very high damping, the branches which correspond to spin-wave excitation and electromagnetic excitation in the lossless insulator limit retain their identities. For very small damping, the branch which is LS (spin-wave) in character at low $|\Omega|$ converts to LE (electromagnetic) at high $|\Omega|$ and vice-versa.

POLARIZATION CHARACTER

The polarizations of the excitations to which the four branches correspond are also of some interest. In fact, the "Larmor" and "antilarmor" labels for the branches are related to the polarizations of the excitations connected with them. The simplest cases for examination are the perpendicular and parallel orientations, in the lossless insulator limit, $L = 0$ and $\varepsilon = 0$. For perpendicular ($\alpha_1 = \alpha_3 = 0$, $\alpha_2 = 1$), the simultaneous equations for the microwave $\boldsymbol{m}$ components m_x and m_z obtained from the $\boldsymbol{A}$ matrix become:

$$\begin{bmatrix} \mathrm{i}|\Omega| & (-\eta + K^2) \\ (\eta - K^2) & \mathrm{i}|\Omega| \end{bmatrix} \cdot \begin{bmatrix} m_x \\ m_z \end{bmatrix} = 0 \,. \tag{14}$$

The nontrivial K^2 solutions, $K^2 = \eta \pm |\Omega|$, correspond to the AS and LS branches previously discussed as shown in Fig. 1. For the AS branch, one obtains $m_z = -\mathrm{i}m_x$ corresponding to antilarmor precession in the sense opposite to the natural precession for electron magnetic moments. For the LS branch, one obtains $m_z = +\mathrm{i}m_x$, corresponding to Larmor precession. In both cases, the modes are circularly polarized, as expected from the problem symmetry.

In the parallel case ($\alpha_1 = 1$, $\alpha_2 = \alpha_3 = 0$) with M_0 along the x-axis parallel to the sample surface, the $\boldsymbol{m}$ equations from the $\boldsymbol{A}$ matrix become,

$$\begin{bmatrix} i|\Omega| & (\eta - K^2) \\ (-\eta - 1 + K^2) & i|\Omega| \end{bmatrix} \cdot \begin{bmatrix} m_y \\ m_z \end{bmatrix} = 0 . \tag{15}$$

Since $|\boldsymbol{A}| = 0$ is a requirement for non-zero $\boldsymbol{m}$, the m_z/m_y ratio can be specified by,

$$\frac{m_z}{m_y} = \frac{-i|\Omega|}{\eta - K^2} = \frac{\eta + 1 - K^2}{i|\Omega|} . \tag{16}$$

The last two terms yield the previously derived quadratic in K^2 for the two spin-wave branches, Eq. (12b) and Fig. 1(b). The two solutions for K^2 when substituted into the above, yield:

$$m_z/m_y = 2i|\Omega|/(1 \pm \sqrt{[4|\Omega|^2 + 1]}) . \tag{17}$$

The $\pm$ signs correspond to the polarizations for the LS and AS branches, respectively. The sense of the precession in the LS (AS) case is the same (opposite) as that for the natural electron moment in an x-directed static field; hence, the initial label Larmor (antilarmor). The precession cones, however, are highly elliptical in both cases. The ellipticities are identical for the LS and AS branches, except that the major axes are orthogonal. In the LS case, the ratio of the in-plane to out-of-plane microwave $\boldsymbol{m}$ components ranges from unity (circular polarization) for $|\Omega| \gg 1$ to $|\Omega|^{-1}$ in the limit $|\Omega| \ll 1$ (highly elliptical with the major axis in-plane). This highly elliptical polarization in the low frequency limit is intuitively reasonable, since demagnetizing effects tend to "squash" the magnetization cone. In the antilarmor (AS) case, the major axis is normal to the metal surface. It is hard to understand this kind of precession cone "squashing" on the basis of intuitive demagnetizing field arguments. On the other hand, a major axis normal to the plane can be seen to satisfy the general requirement that the LS and AS modes be orthogonal. For permalloy at x-band (10 GHz), we have $|\Omega| = 0{\cdot}340$, so that the ellipticity is 0·31.

The polarization and precession sense are also easily evaluated for a general magnetization direction. For the magnetization in the xy plane with its direction specified by the direction cosine α_1($\alpha_1 = 0$ for perpendicular and $\alpha_1 = 1$ for parallel), the result is

$$m_\theta/m_\phi = \pm 2i|\Omega|/[(4|\Omega|^2 + \alpha_1^2)^{1/2} - \alpha_1^2] , \tag{18}$$

where $m_{\theta,\phi}$ represents the magnetization components in the direction of unit vectors for a conventional (r, θ, ϕ) right hand coordinate system referenced to the film (x, y, z) coordinate system in the usual way. The upper sign corresponds to the AS

branch and the lower sign is for the LS branch. In the perpendicular limit, α_1 is zero and the polarizations are circular. In the parallel limit ($\alpha_1 = 1$), elliptical polarizations result. Similar results can be easily obtained for the electromagnetic branches.

SUMMARY

In summary, this paper has presented a calculation of spin-wave dispersion for the allowed normal modes of excitation in a ferromagnetic metal for incident plane-wave microwave excitation to the planar metal surface. The effect of the exchange field on the damping term in the equation of motion has been considered. The secular determinant for the normal modes which results is in a much simplified form, compared to previous results where the exchange contribution to the damping is neglected. The behaviour of the four dispersion branches has been examined. For the damping below a critical value ($\alpha = 0{\cdot}013-0{\cdot}014$ for permalloy parameters), the branches exhibit a crossover character in that the Larmor spin-wave branch at low frequency converts to the electromagnetic branch at high frequency. The low frequency electromagnetic branch (Larmor) converts to a spin-wave character at high frequency. The polarizations of the excitations depend on the field orientation relative to the metal surface. At parallel, the polarizations are elliptical and the major axes are parallel and perpendicular to the metal surface for the Larmor and antilarmor branches, respectively.

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References

[1] AMENT W. S., RADO G. T., Phys. Rev. *97* (1955), 1558.
[2] SOOHOO R. F., Phys. Rev. *120* (1960), 1978.
[3] VITTORIA C., Thesis, Yale University, 1970.
VITTORIA C., BARKER R. C., YELON A., J. Appl. Phys. *42* (1971), 1734.
VITTORIA C., BAILEY G. C., BARKER R. C., YELON A., Phys. Rev. *B 7* (1973), 2112; *8* (1973), 2383.
[4] WILTS C. H., RAMER G., private communication.
[5] AKHIEZER A. I., Soviet Physics Uspekhi *72* (1961), 661.
[6] HEINRICH B., MESHCHERYAKOV V. F., JETP Lett. *9* (1969), 378.