Nonlinear Domain Wall Motion in Magnetic Thin Films

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Abstract — A calculation of the wall-motion response in thin films which combines Walker's dynamic theory with a conservative wall coercive force interaction has been done. The results are compared with recent wall motion data for Permalloy films. The comparison shows that the observed nonlinear response cannot be explained by these two factors alone. A more sophisticated analysis of wall motion in a conservative potential is needed. Relaxation effects may provide an alternate explanation of the nonlinear behavior.

The origin of the observed nonlinear domain wall-motion in thin films has been a perplexing problem for a number of years. The effect was first observed in the quasi-static mobility experiments of Middelhoek [1] and Patton [2]. Several explanations of the nonlinear behavior have been proposed since that time. In his original paper, Middelhoek suggested that due to the rapid rotation of the spins within the domain wall moving at high velocities, the viscous flow approximation might be no longer valid and that such an effect might be the origin of the nonlinear response. Quite recently, Konishi [3] and Patton [4] have applied this idea to wall motion in Permalloy films with qualitative success. Shortly after the nonlinear response was reported, a quite different explanation was proposed by Feldtkeller [5], who was the first to realize the potential importance of coercive force interactions [6], [7] and wall contraction [8] in the context of wall motion for Permalloy thin films. Feldtkeller found that coercive force interactions modified the theoretical wall velocity versus field curves in such a way as to be in qualitative agreement with the data of Middlehoek and Patton. He proposed that the agreement could be improved further by including wall contraction in the analysis.

The validity of this conceptually simple coercive force model to explain the nonlinear wall motion in Ni-Fe films has been tacitly assumed in a number of subsequent interpretations of wall motion data [9], [10], even though no quantitative comparison of the theoretical response due to the combined effect of wall contraction and a conservative coercive force interaction with velocity data has ever been made. The purpose of this paper is to make such a comparison. The results show that these effects alone do not adequately explain the observed nonlinear behavior.

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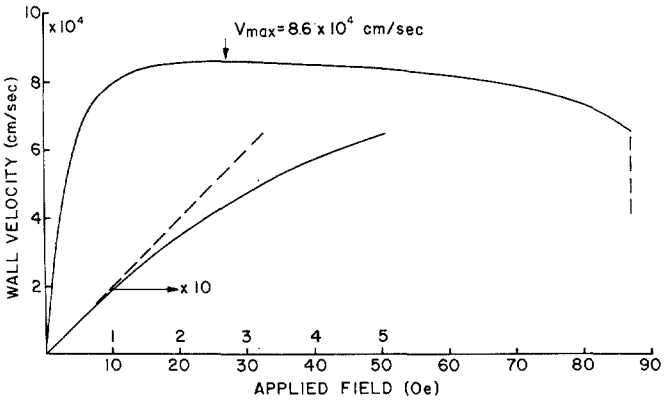

Fig. 1. Domain wall velocity versus field, from the Walker theory [8], for Permalloy films. Material parameters are given in the text.

I. WALL CONTRACTION

The effect of wall distortion on the response follows directly from the wall motion analysis of Walker [8]

$$v_{\text{wall}} = \frac{(\gamma/\alpha)\sqrt{A/K}}{\left\{1+\pi M^2/K\left[1-\sqrt{1-H^2/4\pi^2M^2\alpha^2}\right]\right\}^{1/2}} \cdot H \qquad (1)$$

for a bulk uniaxial material characterized by an exchange parameter A, a uniaxial anisotropy K, a saturation induction of $4\pi M$, a Landau-Lifshitz damping parameter α, and with a drive field H. Gamma (γ) is the gyromagnetic ratio. This basic result has drawn considerable attention recently in relation to both bubble dynamics [11], and wall motion in Permalloy [10], [12]. In Fig. 1, the Walker equation (eq. (1)), is plotted versus easy axis field for typical Permalloy parameters, $4\pi M = 10^4$ G, $H_A = 2K/M = 7$ Oe, $A = 10^6$ erg/cm, and $\alpha = 0.018$. The shape of the total curve has been discussed by Bourne and co-workers [10], [12]. Of particular concern here is the departure of the curve from the Landau-Lifshitz linear response at low drives, shown on the expanded scale (the dotted line shows the Landau-Liftshitz linear response).

The Walker curve bends away from the linear response because of the wall contraction which must occur for wall motion to proceed. This is well known. What has generally not been appreciated in the context of wall motion in Permalloy films is the extent of the contraction for even low drive fields. Figure 1 shows that for a drive of only a few Oe, the contraction results in a velocity 25% lower than expected from the linear response. In past work on Permalloy, the contraction has generally been neglected at low drives, presumably because it was believed that the large $4\pi M$ made

only a small tipping of the moment out of its static rotation plane necessary to produce the wall motion, and that a small tipping implied a small negligible contraction. It is precisely because $4\pi M$ is large that even the small contraction for wall motion produces a large demagnetizing energy contribution, and hence a large contraction.

II. COERCIVE FORCE INTERACTIONS

The Walker theory deals specifically with the limit of a zero wall motion coercive force. For wall motion in Permalloy films, the observed coercive force is generally on the order of one Oe or greater, so that velocity field curves fall to $V = 0$ at $H - H_c$, not at $H = 0$. In the context of wall motion in bulk materials, it has been realized for some time that coercive force interactions can do more than simply shift the zero velocity intercept to H_c. The detailed physical processes which give rise to H_c also significantly affect the shape of the velocity curve for $H > H_c$.

Following the discussions of Rodbell [6] and Baldwin [7], two distinct limiting cases may be considered. First, for a completely dissipative or *nonconservative* process, the coercive force interaction causes energy dissipation at a rate $2MH_cV$ during wall motion. In this case, only the field difference $(H - H_c)$ will serve to drive the wall, and the Walker solution may be taken over simply by replacing H by $H - H_c$. Velocity curves would then look exactly the same as before, but with the $V = 0$ intercept at $H = H_c$. Second, for a completely nondissipative or *conservative* coercive force interaction, the coercive force interaction causes no energy dissipation during wall motion. The wall motion under these circumstances is crudely analogous to a particle moving in a conservative potential distribution. A minimum drive level is required to drive the wall over the peaks in the potential, but no energy is dissipated by the motion. For a given drive field, $H > H_c$, the wall will simply slow down as it goes "up" one side of each hump in the potential and speed up as it slides "down" the other side.

As pointed out by both Rodbell and Baldwin, wall motion in such a conservative potential will lead to a nonlinear response. This can be seen by examining wall motion where H barely exceeds H_c. When the wall reaches the "tops" of the potential, it will be moving extremely slowly. These periods of very slow motion will have a dominant effect on the average velocity. For $H \gg H_c$, the wall kinetic energy will be much greater than any of the potential barriers, and their effect on its motion will be negligible. Feldtkeller incorporated such considerations into the calculation of the wall response by viewing the wall potential in terms of a position dependent driving field $H_f(x)$ which ranges $-H_c < H_f < H_c$. The total effective field driving the wall is given by $H = H_0 + H_f(x)$, where H_0 is the applied field. Translational motion is possible only if H_0 is greater than H_c. Feldtkeller applied this result to the linear wall motion equation by writing the instaneous position dependent velocity

$$V(x) = dx/dt = G_0\ [H_0 + H_f(x)] \tag{2}$$

and calculating the *average* wall velocity

$$V = dx/\int dx/G_0\ [H_0 + H_v(x)]. \tag{3}$$

The integration over distance was conveniently eliminated by describing the distribution of H_f as a probability function $p(H_f)$, so that the final velocity expression becomes

$$V = \left[\int_{-H_c}^{+H_c} \frac{p(H_f)\,dH_f}{G_0(H_0 + H_f)}\right]^{-1} \tag{4}$$

subject to $H_0 > H_c$.

Feldtkeller applied this analysis to a number of simple coercive force distributions and showed that a nonlinear wall motion response similar to that observed experimentally could result. As seen in the previous section, however, the wall contraction alone results in a significant nonlinear response. In order to make a realistic comparison with experiment, it is necessary to include both effects in the analysis simultaneously. This may be easily accomplished by using the Walker wall velocity expression of Eq. (1) rather than the linear response. The required expression for the average velocity V becomes

$$V = (\gamma/\alpha)\sqrt{A/K}\left[\int_{-H_c}^{+H_c} \frac{p(H_f)}{H_0 + H_f}\left\{1 + \pi M^2/K\left[1 - \sqrt{1-(H_0 + H_f)^2/(2\pi M\alpha)^2}\right]\right\} dH_f\right]^{-1} \tag{5}$$

subject to conditions, $H_0 > H_c$ and $(H_0 + H_c)/2\pi\alpha M < 1$. By utilizing different coercive force distributions $p(H_f)$, explicit velocity curves may be generated for comparison with wall motion data. Three specific cases have been considered: 1) a parabolic potential, so that $H_f(x)$ is a periodic triangular function, and $p(H_f) = 1/2H_c$ for $-H_c < H_f < H_c$; 2) a spike distribution with $H_f = \pm H_c$; and 3) a sinusoidal potential. The distribution functions for these three cases are as follows.

Case 1) Uniform Distribution:

$$p(H_f) = (2H_c)^{-1}, \qquad |H_f| < H_c. \tag{6}$$

Case 2) Spike Distribution:

$$p(H_f) = \frac{1}{2}\left[\delta(H_f - H_c) + \delta(H_f + H_c)\right]. \tag{7}$$

Case 3) Sinusoidal Potential:

$$p(H_f) = \frac{1}{\pi H_c}\,\frac{1}{\sqrt{1-(H_f/H_c)^2}}, \qquad |H_f| < H_c. \tag{8}$$

Velocity field curves were calculated for these three distributions (using typical parameters for Permalloy films) by means of a digital computer. Numerical integration subroutines were utilized in cases 1) and 3).

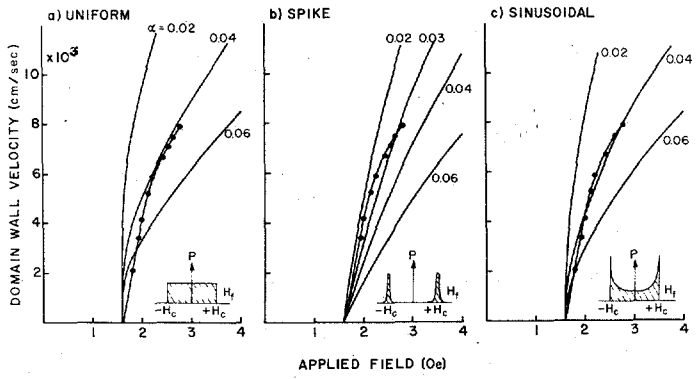

Fig. 2. Calculated velocity-field curves for different distributions and damping values, compared with data for a 310 Å film (data from ref. 10).

III. RESULTS AND DISCUSSION

The results of the computer calculations are shown in Fig. 2. The curves were obtained for $H_c = 1.6$ Oe, $H_A = 3.5$ Oe, $4\pi M = 10^4$ G, $A = 10^{-6}$ erg/cm, and different values of the damping parameter, α, as indicated. These parameters were chosen to provide a good comparison with the data of Konishi on a 310 Å film [13]. Several comments are in order to justify the comparison of data for such a thin film, where the domain wall is essentially of a Néel character, sparsely populated with cross ties, when the Walker theory was strictly applicable to a one-dimensional Bloch wall configuration for an infinite medium.

As evident from the work of Middelhoek [14], among others, it is clear that a Néel wall in the limit of zero thickness is energetically equivalent to a Bloch wall in bulk. This basic result is also evident from the formal treatment in Walker's theory. If the wall distortion angle ϕ in Walker's theory is referenced to the film normal instead of the wall normal ($\phi \approx \pi/2$ at low drive) the spin rotation corresponds to that for a Néel wall. In the limit of zero thickness, the analysis is exactly the same as given by Walker. For a 300 Å film thickness, the actual Néel wall is contracted statically due to demagnetizing energy, and it is necessary to replace the wall width in the Walker theory, $a_o = \pi\sqrt{A/K} \sim 7500$ Å, by an empirical width as determined by Lorentz microscopy. From the work of Suzuki and others [15], the wall width at 300 Å thickness can be expected to range from 1000 Å to 4000 Å, depending on the amount of spin rotation within the wall "core" region. For the sake of analysis, a value of 2500 Å from earlier data [16] was used. From [15], this would correspond to a spin rotation in the wall core of about 140–150°, consistent with the observations of Fuchs [17]. No rigorous justification of this empirical adjustment to the Walker theory is argued. It does render the theory equivalent to the linear theory in the limit of zero drive, and should provide a reasonable prediction of the nonlinear response at low drive levels.

Also of concern is the influence of the cross ties on the wall motion response. If they affect the dynamics in any significant way, an analysis for a homogeneous wall structure is inappropriate. There has been considerable discussion of this point in the literature. Shishkov [18] found that the wall velocity is reduced by the presence of cross ties, with the amount of reduction proportional to the cross-tie density. He also found, however, that below about 400 Å thickness, the cross ties are so sparse that they have little or no effect on the wall response.

The one remaining possible problem is in the basic validity of the one-dimensional wall model in the first place. Hubert [19] and others have proposed two-dimensional spin distributions which yield lower static wall energies than do one-dimensional distributions in many cases. Indeed, this has been a troublesome point in the interpretation of Bloch wall motion data for thicker films [10]. It has been necessary [10] to introduce an empirical parameter to account for the flux closure in two-dimensional Bloch walls which makes the distortion less effective in driving the wall than in the Walker theory. Very thin films, however, are ideally suited for the avoidance of these problems. Hubert has found [19] that for film thickness less than about 500 Å, a one-dimensional structure is an extremely good approximation for the spin distribution within the wall.

Thus it appears that Néel wall motion for very thin films represents a nearly ideal situation for analysis. 1) The Walker theory, with a simple wall width adjustment, is applicable to at least first order. 2) Cross ties do not significantly affect the motion. 3) The wall is essentially one dimensional and exhibits little or none of the flux closure complications evident in thicker films.

Figure 2a, b, and c shows calculated velocity curves for the uniform H_c distribution, the spike distribution, and the sinusoidal distribution, respectively. Similar characteristics are found in all cases, with slight modifications for the different distributions used. The results show, in addition, the rather poor agreement between the calculated curves and velocity data. While both data and the theory show about the same degree of nonlinearity, no combination of parameters can yield a quantitative fit over the entire field range of the data. If α is adjusted to match the data for H near H_c, as for the spike distribution and $\alpha = 0.02$, the results do not match up a higher fields. If the curves match up at higher fields, as for the uniform distribution and $\alpha = 0.04$, the fit is extremely poor at low field. The best overall fit is for the sinusoidal distribution and $\alpha = 0.04$, although the theoretical curve is still much too steep at low field. These comparisons are representative of those found for other data on films of different thicknesses.

One particular result of the comparison in Fig. 2, which is quite disturbing, is the rather large value of the damping ($\alpha \sim 0.04$) which is needed to obtain the "best fit" between theory and data. This value is much larger than those inferred from previous wall motion experiments ($\alpha \sim 0.01$). It appears unlikely that the wall motion damping is so big, especially in light of the good agreement found previously with predictions based on wall width determinations and $\alpha \sim 0.01$ [2], [16], [18] and the observed correlation between wall motion and ferromagnetic resonance in thin films [20].

What conclusions can be drawn from these comparisons? In the author's opinion, the results provide a very strong indica-

cation that the effects of a conservative wall motion potential and wall contraction alone cannot account for the observed nonlinear wall motion. By some standards, namely the large errors in the wall motion data of several years ago, the data and theory might be considered to be in agreement. In recent years, however, Telesnin and co-workers [18] and Konishi [3], [13] have refined the quisi-static wall mobility techniques to attain high accuracy and good reproducibility. Hence, the above conclusion is justified.

The validity of this conclusion hinges on the applicability of the Walker theory, with the described alterations, and the simple conservative coercive force model to the data at hand. As already discussed, the Walker analysis along with related considerations of homogeneous one-dimensional walls, etc., seems to be entirely applicable, at least to first order. Since we are essentially comparing the shapes of the curves by adjusting the damping, the approach appears to be respectable.

The simple conservative coercive force model, on the other hand, may be subject to considerable debate. In terms of the basic wall interactions with inclusions, pits, etc., in thin films, a nonconservative interaction might be much more plausible. The present results certainly point in this direction. A nonconservative interaction, however, does not contribute to the nonlinear response, and wall contraction alone is inadequate to account for the observed nonlinear behavior. An alternate mechanism for the nonlinear response may arise from relaxation effects [1], [3], [4]. This mechanism needs to be examined in a quantitative manner.

Before the conservative interaction model is completely rejected, however, a more sophisticated analysis of its implications concerning wall motion should be undertaken. In the present analysis, the approach was essentially quasi-static, in which the wall was assumed to adjust instantaneously to its local field environment. From the recent quasi-static velocity experiments with pulse lengths down to 35 nsec and velocities in the 10^3-10^4 cm/sec range [10], it seems that the average periodicity of any conservative potential which might be present is quite small, on the order of a few thousand Å or less. A wall will move this distance in 10 nsec or less, barely time to adjust instantaneously to the local field environment along its path. Even more important, the above indicates that the wall itself is as wide or wider than the wavelength of the conservative potential. Clearly, a more sophisticated analysis is required to treat this situation.

IV. CONCLUSIONS

In summary, the following points should be recounted. 1) Néel wall motion in very thin films ($<$ 300 Å) represents a highly suitable situation for the application of one-dimensional wall dynamics in the analysis of the response. 2) The simple Feldtkeller conservative coercive force analysis, in combination with the Walker theory, suitably adjusted for the actual wall width, does not adequately account for the observed response. 3) If the conservative model is rejected, wall contraction alone is inadequate to explain the nonlinear response. Relaxation effects have been suggested as an alternative mechanism, and a quantitative analysis of such processes is needed. 4) The success of quasi-static wall velocity experiments with nanosecond field pulses suggests that the effect of conservative potential on wall dynamics may be more subtle than that given by the Feldtkeller approach, and that a more sophisticated treatment of this mechanism is also needed before any truly definitive judgment can be made.

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