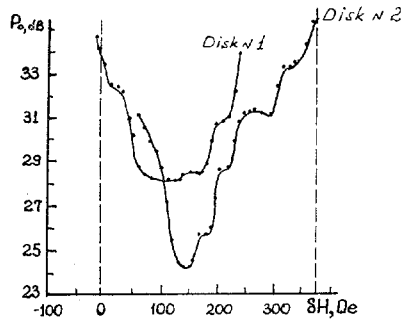

Fig. 1.

type of oscillation well agree with the experimental one, only with the assumption that disks are hard fastened by the perimeter.

By means of the method of construction of isoenergy surfaces [2], it is shown, that on two-stage mechanism excitation of MAR, the wave number of pumping wave is governed by the equation

$$k \leq \frac{\omega_M}{2v_a} = k_{\max}$$

(where v_a is the velocity of the elastic waves). For YIG the values of $k_{\max}$ are equal to 10^4 cm^{-1}, that is on two-stage MAR the "second" pumping wave is not an exchange spin wave, but a magnetostatic one. The region of the static magnetic field, where the two-stage MAR can exist, has the following limits

$$N_z N_s - 2\pi M_s + \sqrt{(2\pi M_s)^2 + (\omega/\gamma)^2} \leq H_0 \leq \omega/\gamma + N_z M_s$$

where N_z is the demagnetizing factor in the direction of the static external magnetic field H_0. In Fig. 1 the theoretical limits of MAR existing are shown by dotted lines. A good accordance with experimental data takes place.

References

[1] V. D. Burkov, A. V. Vashkovsky, and V. N. Kildishev, *Radiotekh. Elektron.*, vol. 15, no. 4, 1970, p. 767.
[2] F. V. Lisovsky, *Radiotekh. Elektron.*, vol. 14, no. 8, 1969, p. 1511.

Direct Evidence for a Two-Magnon Contribution to the FMR Relaxation in Ni–Fe Thin Films

CARL E. PATTON, SENIOR MEMBER, IEEE, FUMIHISA ONO, AND MINORU TAKAHASHI

Abstract—An off-resonance technique, which permits the relaxation rate to be determined as a function of bias field, has been applied to Ni–Fe thin films for the first time. The data provide direct evidence for a strong two-magnon scattering contribution to the losses. The effective linewidth for a 1300-Å film at 9 GHz peaks near the parallel resonance position with $\Delta(\omega/\gamma)_{\text{eff}} \backsimeq 200$ Oe. It decreases above and below the ferromagnetic resonance (FMR). $\Delta(\omega/\gamma)_{\text{eff}}$ is reduced to 60 Oe at a bias 100 Oe below resonance. This sharp falloff indicates that the scattering is due to relatively large inhomogeneities, 3000 Å or larger in size. The falloff above resonance is due to a decrease in both the density of states and coupling for the degenerate spin waves. The effective line shift due to spin-wave scattering is also strongly field dependent. It changes by more than 30 Oe in the vicinity of resonance. The data show that line-shift effects should completely obscure ripple field shifts (0.1–1 Oe) which can, in concept, be measured by FMR techniques.

ON THE BASIS of linewidth data, two-magnon scattering has been proposed as one major contribution to relaxation for both standing spin-wave modes [1], [2] and uniform precession resonance [3] in evaporated thin films. In this investigation an off-resonance technique, which permits the relaxation rate to be determined as a function of bias field [4], has been applied to thin films for the first time. The results provide the first direct evidence for a strong two-magnon scattering contribution to the losses. The basic technique is to measure both the real and imaginary parts of the susceptibility at a given bias field and then use these data to calculate the relaxation rate η from a uniform precession response analysis. This relaxation rate η is usually expressed as an effective linewidth $\Delta(\omega/\gamma)_{\text{eff}} = 2\eta/\gamma$ for comparison with conventional linewidth data. In a plot of $\Delta(\omega/\gamma)_{\text{eff}}$ versus magnetic field, any two-magnon contribution shows up directly because of the characteristic field dependence of the spin-wave density of states and the two-magnon coupling strength at the drive frequency.

Data were obtained on evaporated 80–20 Ni–Fe films 900–1500 Å thick for both parallel and perpendicular resonance at 9.2 GHz and room temperature. The microwave spectrometer was a standard reflection type utilizing a rectangular TE_{102} cavity and a klystron source. The klystron was stabilized on the cavity frequency using a simple frequency-modulation automatic frequency control (AFC) scheme due to Berry and Benton [5]. The raw data appear in the form of an absorption curve of crystal output versus bias field and a dispersion curve of AFC feedback voltage versus bias. By carefully calibrating the crystal

Manuscript received February 26, 1971; revised April 23, 1971. Paper 31.4, presented at the 1971 INTERMAG Conference, Denver, Colo., April 13–16. This paper was supported in part by the Japan Society for the Promotion of Science.

C. E. Patton was with Tohoku University, Sendai, Japan, on leave from the Research Division, Raytheon Company, Waltham, Mass.

F. Ono and M. Takahashi are with the Department of Applied Physics, Faculty of Engineering, University of Tohoku, Sendai, Japan.

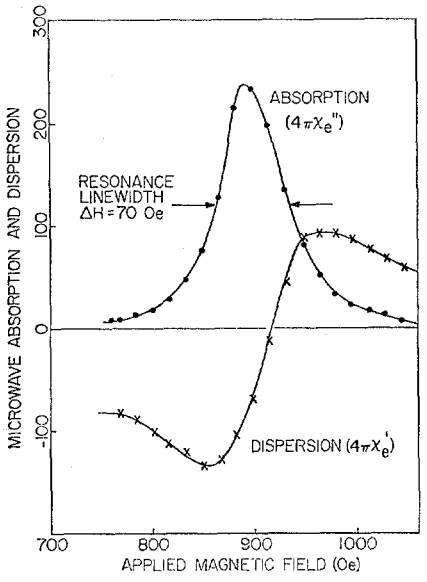

Fig. 1. Microwave absorption and dispersion curves for parallel resonance in 1277-Å film at 9.2 GHz. Numbers on ordinate axis refer to $4\pi\chi_e''$ and $4\pi\chi_e'$ for absorption and dispersion, respectively.

output curve in terms of microwave power and the AFC voltage in terms of cavity frequency shift, true absorption χ_e'' (negative imaginary part of the external susceptibility) and dispersion (real part of the external susceptibility χ_e') curves can be obtained using standard microwave perturbation theory.

Absorption and dispersion curves are shown in Fig. 1 for a 1277-Å thick film. The half-power linewidth, the half-width of the absorption curve, is about 70 Oe. In most resonance studies, this single loss parameter is used to characterize the microwave relaxation. In cases where the relaxation mechanisms may be field dependent, as for two-magnon scattering, a single loss parameter is inadequate. By including dispersion data in the analysis, it is possible to determine the field dependence of the losses in the form of a relaxation rate η or effective linewidth $\Delta(\omega/\gamma)_{\text{eff}} = 2\eta/\gamma$ versus magnetic field.

The analysis to obtain η or $\Delta(\omega/\gamma)_{\text{eff}}$ from external or effective susceptibility data is based on the uniform precession response. From the Bloch–Bloembergen formulation, the near resonance effective susceptibility is given by

$$4\pi\chi_e = \frac{\omega_m\omega_y}{\omega_x\omega_y - \omega^2 + 2i\omega\eta} \tag{1}$$

where

$$\omega_m = \gamma 4\pi M_s$$

$$\omega_{x,y} = \gamma[H_0 + (N_{x,y} - N_z)4\pi M_s] = \gamma W_{x,y}.$$

γ is the electron gyromagnetic ratio (taken as positive), $4\pi M_s$ is the material saturation induction, H_0 is the applied external field in the z direction, and $N_{x,y,z}$ are the sample demagnetizing factors with the microwave field along the x-directed principal axis. The real and imaginary parts of (1) constitute two equations for η and γ. All other parameters are specified by the material or the experimental

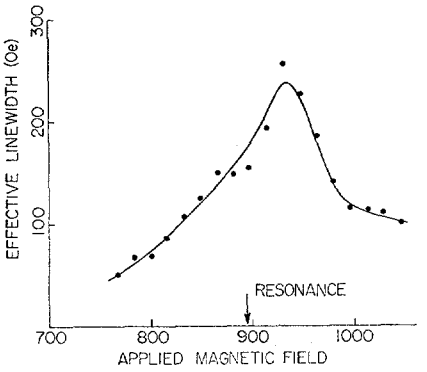

Fig. 2. Effective linewidth versus static field derived from data in Fig. 1.

configuration. The appropriate value of $4\pi M_s$ was estimated from the parallel and perpendicular resonance positions. The working equations are

$$\eta = \frac{1}{2}\frac{\gamma 4\pi M_s \gamma W_y}{\omega\, 4\pi\chi_e''}\frac{1}{1 + (\chi_e'/\chi_e'')^2} \tag{2a}$$

$$\gamma^2 = \frac{\omega[(\chi_e'/\chi_e'')2\eta + \omega]}{W_x W_y}. \tag{2b}$$

Computer solutions for η and γ are easily obtained by an iterative technique. In the first iteration, γ is assumed equal to the free electron value (1.76×10^7 Oe^{-1} s^{-1}) and η is calculated from (2a). This value of η is then used in (2b) to get a better estimate for γ. The sequence is repeated until the change from one cycle to the next is negligibly small. In this analysis, the iteration was arbitrarily terminated when η changed by less than 0.1 percent. With η and γ determined in this way, an effective linewidth parameter

$$\Delta\left(\frac{\omega}{\gamma}\right)_{\text{eff}} = \frac{2\eta}{\gamma} \tag{3}$$

is obtained at the particular bias field H_0 of interest. Note that the effective linewidth is expressed as a frequency-swept linewidth. The field-swept effective linewidth for the frequency of maximum reliability (FMR) in films contains an additional geometry dependent factor $2\omega/(\omega_x + \omega_y)$, as discussed in [3] and [6].

The data in Fig. 1 were analyzed to obtain $\Delta(\omega/\gamma)_{\text{eff}}$ versus applied field. The results are shown in Fig. 2. It should be emphasized that the curve in Fig. 2 is *not* a resonance curve. It is an actual plot of the uniform precession relaxation rate, in linewidth units, versus magnetic field. Similar parallel resonance results were obtained for all films in the thickness range examined. $\Delta(\omega/\gamma)_{\text{eff}}$ is maximum near the resonance position and decreases as the bias is either decreased or increased. It is clear from these data that the FMR losses in films are strongly field dependent and that a single linewidth parameter is completely inadequate.

Notice that the effective linewidth $\Delta(\omega/\gamma)_{\text{eff}}$ in the vicinity of resonance is about 200 Oe, while the absorption curve half-power linewidth ΔH in Fig. 1 is only 70 Oe. Data for ferrites have generally given similar values for ΔH and the near resonance effective linewidth. The reason for

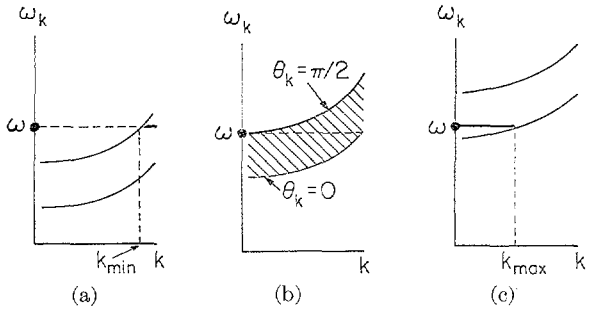

Fig. 3. Spin-wave manifold positions relative to excitation frequency for bias fields in thin film. (a) Below resonance. (b) Parallel resonance. (c) Above resonance.

the large difference here is the geometry-dependent factor $2\omega/(\omega_x + \omega_y)$ in the relation between ΔH and $\Delta(\omega/\gamma)$. For the present experimental conditions, this factor is about 0.3, so that $\Delta H = 70$ Oe corresponds to a frequency-swept linewidth $\Delta(\omega/\gamma)$ of 230 Oe. This value is fairly close to the measured $\Delta(\omega/\gamma)_{\text{eff}}$ near resonance.

The remaining discrepancy can be attributed to effective line shift. The same spin-wave interactions which cause η, the imaginary part of the uniform mode frequency, to vary with field cause a change in the real part as well. This change shows up as a deviation of γ from the intrinsic value γ_0 in the vicinity of resonance. The change is rather small, on the order of a few percent in most cases. Just as $\Delta(\omega/\gamma)_{\text{eff}}$ puts η into linewidth units, the deviation of γ from γ_0 can be expressed as an effective line shift, according to

$$\gamma^2 W_x(H_0) W_y(H_0) = \gamma_0^2 W_x(H_0 + S_{\text{eff}}) W_y(H_0 + S_{\text{eff}}). \tag{4}$$

For parallel resonance at 9 GHz, one obtains $S_{\text{eff}} \approx H_0 \cdot (\gamma^2/\gamma_0^2 - 1)$. With no reliable independent way to get γ_0 and experimental errors of at least one percent for γ, it is impossible to obtain quantitative results for S_{eff} versus field. The several percent change in γ around resonance however, does suggest S_{eff} changes on the order of 30 Oe or so. These shifts would certainly affect the observed linewidth. These shifts should be important especially at low frequencies, where FMR techniques have been used to study the internal ripple fields in thin films [7]. Our data indicate that line-shift effects should completely obscure any ripple field shifts (0.1–1.0 Oe) which are present.

The nature of the dependence shown in Fig. 2 provides the first evidence for a two-magnon scattering contribution to thin-film FMR losses. Only this kind of mechanism can give the strong field dependences in the vicinity of resonance found in the present study. In order to understand clearly the connection, consider for a moment the shift in the spin-wave spectrum as the field is swept through FMR. Fig. 3 shows the position of the spin-wave manifold relative to the excitation frequency for bias fields below resonance, at resonance, and above resonance. At resonance(Fig. 3(b)) ω falls at the $k = 0$ end of the $\theta_k = \pi/2$ branch (θ_k is the angle between the bias and the spin-wave wave vector $\boldsymbol{k}$). Near resonance, therefore, the density of states degenerate with the driving frequency is largest. Consequently, the two-magnon relaxation contribution should be big near the FMR peak. Fig. 2 shows that the effective linewidth is indeed largest in this range, on the order of 200 Oe.

It should be noted that the peak in $\Delta(\omega/\gamma)_{\text{eff}}$ is about 45 Oe above the resonance position as defined by the absorption peak. There is, however, some ambiguity in the definition of resonance position as the zero crossing of $4\pi\chi_e'$ occurs at a slightly higher field. If the FMR position is taken at this point, the discrepancy is only 25 Oe. The ambiguity in the $4\pi M_s$ value used in the analysis is certainly greater than these shifts so that the discrepancy need not be viewed with any great concern.

When the bias falls below resonance, the position of the manifold drops relative to ω, as shown in the left-hand diagram of Fig. 3, and the cutoff wavenumber k_{min} for degenerate modes increases. Only modes with $k > k_{\text{min}}$ are available for scattering. As the density of degenerate states is decreased, $\Delta(\omega/\gamma)_{\text{eff}}$ is also expected to decrease. Fig. 2 shows that $\Delta(\omega/\gamma)_{\text{eff}}$ falls off quite rapidly below resonance. The severity of the falloff, moreover, implies that low-k modes play the most important role in the scattering. At the lower field limit of 760 Oe for the present data, $\Delta(\omega/\gamma)_{\text{eff}} = 60$ Oe is on the order of intrinsic thin-film linewidths [3] with no two-magnon scattering contribution. This bias corresponds to $k_{\text{min}} = 2 \times 10^5$ cm^{-1}. This result implies that modes with $k > 2 \times 10^5$ cm^{-1} do not contribute significantly to the two-magnon relaxation in evaporated films. Strong coupling to modes with $k < 2 \times 10^5$ cm^{-1} suggests inhomogeneity sizes on the order of 3000 Å or greater. The crystallite sizes in evaporated films are generally much smaller (typically 100–500 Å). This result is somewhat surprising. It seems unlikely that pits (or holes) or this size exist in the films. Perhaps substrate topography is involved. The present result can only be taken as an initial clue to the nature of the inhomogeneities which give rise to the scattering relaxation in films.

As the bias increases above the resonance point, the manifold is shifted upward, as shown in the right-hand diagram of Fig. 3, and the upper cutoff wavenumber k_{max} for degenerate modes decreases. Only modes with $k < k_{\text{max}}$ are available for scattering. At the same time, the value of θ_k for low k modes decreases from $\pi/2$. These variations reduce the density of dengenerate states. The reduction in θ_k also decreases the coupling strength between the uniform precession and spin-wave modes. Both of these factors imply a reduction in $\Delta(\omega/\gamma)_{\text{eff}}$ at fields above resonance. The data in Fig. 2 confirm this prediction. Unfortunately, the sensitivity of the system was not adequate to obtain data above about 1050 Oe ($\theta_k \approx 67°$). If the exact shape of the falloff could be measured out to fields where k_{max} and θ_k approach zero, it should be possible to infer the precise nature of the inhomogeneities giving rise to scattering.

The shapes of the absorption curves for perpendicular resonance were qualitatively consistent with the preceding scattering interpretation of the parallel resonance data, in that the curves were generally steeper on the high field

than on the low field side. Since perpendicular resonance occurs when the driving frequency is coincident with the $\theta_k = 0$ branch of the spin-wave manifold at $k = 0$, there are no degenerate states at fields above resonance. Consequently, the absorption falls off more steeply in this region. The sensitivity for the perpendicular configuration, however, was insufficient for precise $\Delta(\omega/\gamma)_{\text{eff}}$ determinations.

Of course, the validity of this type of effective linewidth analysis and the two-magnon interpretation of the results depends critically on the applicability of the uniform precession response condition specified by (1). Appreciable pinning of the surface spins or exchange conductivity broadening would require that the response equation be suitably modified in order for the effective linewidth analysis to be valid. The following remarks are intended to justify, at least qualitatively, the validity of the uniform mode assumption on which the analysis is based. It should be emphasized that, apart from the two-magnon result, the basic objective of the present study was to demonstrate the power of the effective linewidth technique in the study of thin-film microwave relaxation. No extensive study of mode configurations or relaxation has been attempted.

Bajorek and Wilts [8] have recently shown that the uniform mode approximation is generally well satisfied for evaporated films of reasonable quality. Indeed, perpendicular resonance spectra for the present films showed extremely weak spin-wave peaks, which is also indicative of weak or negligible pinning. It should be pointed out, moreover, that pinning has generally been more of a problem for perpendicular than for parallel resonance. In spite of the considerable attention given to pinning in recent years, it appears that in most cases, and in our case in particular, pinning is not a problem [8].

Exchange conductivity effects should also be considered. Rodbell [9] has reported line-shape asymmetry for Ni platelets and Fe whiskers which can be attributed to this type of inhomogeneous broadening and is related to a variation of the microwave penetration depth as one passes through FMR. However, Rodbell also found that exchange conductivity broadening was not important when the platelet (or film) thickness was very small compared to the off-resonance microwave skin depth. Rodbell gave specific data for 800-Å platelets. For 1000-Å films at 9 GHz, where the skin depth (off resonance) is greater than 2 μm, the neglect of exchange conductivity broadening seems entirely justified. Schlömann [10] has calculated the effect of simple conductivity or eddy current broadening for parallel FMR in films. His results indicate that the broadening at 1000 Å would be at most a few oersteds and, more important, that the absorption curves would be symmetric. Consequently, conductivity effects do not appear to be a problem.

Acknowledgment

The authors wish to thank T. Miahara for assistance in preparing the films used in this study, and E. Schlömann and J. H. Van Hook for a critical reading of the manuscript.

References

[1] P. E. Wigen, "Dipole narrowed inhomogeneously broadened lines in ferromagnetic thin films," *Phys. Rev.*, vol. 133, Mar. 1964, pp. A1557–A1562.
[2] C. T. Shirkey and P. E. Wigen, "Temperature dependence of thin-film ferromagnetic resonance linewidths," *Phys. Rev. Lett.*, vol. 24, May 1970, pp. 1024–1026.
[3] C. E. Patton, "Linewidth and relaxation processes for the main resonance in the spin-wave spectra of Ni–Fe alloy films," *J. Appl. Phys.*, vol. 39, June 1968, pp. 3060–3068.
[4] C. E. Patton, "Effective linewidth due to porosity and anisotropy in polycrystalline yttrium iron garnet and Ca–V substituted yttrium iron garnet at 10 GHz," *Phys. Rev.*, vol. 179, Mar. 1969, pp. 352–358.
[5] J. E. Berry and A. Benton, "Self-contained transistorized AFC circuit for a klystron power supply," *Rev. Sci. Instrum.*, vol. 36, July 1965, pp. 958–959.
[6] C. E. Patton, "Microwave relaxation and phenomenological damping in thin films," *Czech. J. Phys.*, May 1971.
[7] J. H. Hoper, "Magnetization-ripple internal fields in thin magnetic films measured by ferromagnetic resonance," *J. Appl. Phys.*, vol. 41, Mar. 1970, pp. 1331–1333.
[8] C. H. Bajorek and C. H. Wilts, "Evidence for the absence of surface spin pinning in ferromagnetic resonance," *J. Appl. Phys.*, to be published.
[9] D. S. Rodbell, "Magnetic resonance in high quality ferromagnetic metal single crystals," *Physics*, vol. 1, May 1965, pp. 279–305.
[10] E. Schlömann, "Theory of linebroadening in metal films," A. F. Mater. Lab., Wright-Patterson AFB, Ohio, Tech. Rep. FFML-TR-67-201, June 1967.