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Influence of Spin-Wave Linewidth on the Instability Threshold for Oblique Pumping

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A number of refinements to the Suhl[1] and Schlömann[2] theories of spin-wave instability thresholds have been reported which extend the analysis to include arbitrary polarizations for the microwave field[3,4] and ellipsoidal samples without rotational symmetry about the magnetization direction.[4] Limited experimental data at a few field points on rare-earth-doped YIG and oblique pumping (excitation with a linearly polarized microwave field at an angle $0° \lesssim \psi \lesssim 90°$ with the static field) confirm the predictions for saturated samples.[5] On the other hand, curves of the threshold h_{crit} versus static field for oblique pumping in single-crystal lithium ferrite have been recently reported which are substantially different from the theory.[6] The purpose of this note is to show that (a) the published theoretical curves are correct if the spin-wave linewidth is independent of wave vector $\mathbf{k}$ and (b) the theory yields predictions which agree with single-crystal data when the $\mathbf{k}$-dependence of ΔH_k is taken into account. From these results, it follows that oblique pump data provide a means of actually determining the $\mathbf{k}$-dependence of the spin-wave linewidth.

Two series of measurements were made, one for 1% Dy-doped polycrystalline YIG and one for pure single-crystal YIG. The ΔH_k for the Dy-doped YIG is essentially independent of $\mathbf{k}$, as shown by the parallel pump data of Ref. 5. The ΔH_k for single-crystal YIG, however, has a strong $\mathbf{k}$-dependence.[7] The present data were obtained using well-known microwave techniques.[8] A magnetron source provided 4-μsec pulses at a 60-Hz rate for the Dy–YIG measurements. A klystron driven TWT supplied 100-μsec pulses for the single-crystal measurements. All data were taken at 9.1 GHz and room temperature.

In Fig. 1 the threshold field h_{crit} determinations are shown as a function of the static external field for the Dy-doped polycrystalline YIG. The angle between the pump field and the static field is denoted by ψ. Butterfly curves (h_{crit} vs static field) were obtained for $\psi = 0°$ (parallel pumping), 30°, 60 , and 90° (prependicular pumping. The change in the shape of the butterfly curves with pump angle is in good agreement with the theory

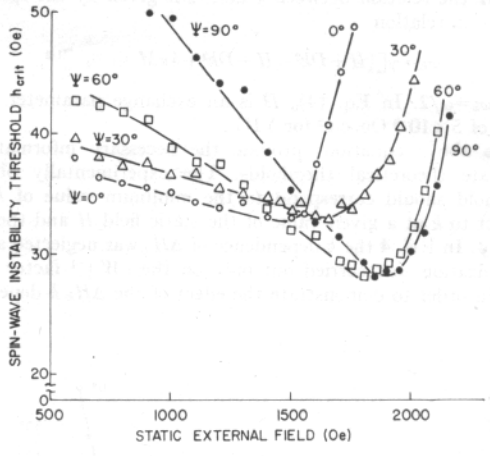

FIG. 1. Curves of the spin-wave instability threshold h_{crit} at 9.1 GHz versus static magnetic field for polycrystalline Dy–YIG. The pump angle between the linearly polarized microwave field and the static field was varied between 0° and 90° in steps of 30°.

(see Fig. 4 of Ref. 4). A smooth transition from the parallel pump curve to the perpendicular pump curve takes place as ψ increases from 0° to 90°. The curves exhibit only a single minimum, no matter what the pump angle. As discussed in Ref. 4, the theoretical curves were obtained by neglecting the k-dependence of ΔH_k. Small discrepancies between the theoretical curves and the data in Fig. 1, such as the nonzero slope for the low field part of the $\psi = 0°$ data, is due to the small $\mathbf{k}$-dependent part of ΔH_k for the Dy–YIG. In general, however, the data show that the theoretical curves are correct in the limit of a $\mathbf{k}$-independent ΔH_k.

The data for single-crystal YIG are shown in Fig. 2. These results are quite different from the Dy–YIG curves. In addition to the expected reduction in the magnitude of h_{crit} for the single crystal, the shapes of the curves are completely different. No smooth transition from parallel to perpendicular pumping occurs as ψ is increased from 0 to 90°. Instead, the butterfly curves at intermediate angles exhibit some of the characteristics of the curves at both extremes. The data for $\psi=60°$, for example, has two local minima, one near the position of the parallel pump ($\psi=0°$) minimum and one near the perpendicular pump ($\psi=90°$) minimum. These results deviate substantially from the data in Fig. 1 for Dy-doped YIG and the theoretical curves in Ref. 4.

The differences in the oblique pumping results in the two cases appear to be due to the influence of the k-dependence of ΔH_k on h_{crit} for the single-crystal YIG. In order to demonstrate this dependence explicitly, new theoretical curves have been evaluated which take this dependence into account. From Ref. 4, the threshold for any mode $\mathbf{k}$ is given by

$$h_c=(\omega/\omega_m)\,(\Delta H_k/|\,W\,|), \tag{1}$$

with $|\,W\,|$ defined by

$$|\,W\,|=\sin\theta_k\left(\frac{\cos\theta_k\sin\psi}{\omega^2-\omega_{res}^2}\,[2\omega_0 f(\theta_k)+\omega^2]+\cos\psi\,\sin\theta_k\right) \tag{2}$$

for oblique pumping with a linearly polarized microwave field. In Eq. (2)

$$f(\theta_k)=\tfrac{1}{2}[(\omega_m^2\sin^4\theta_k+\omega^2)^{1/2}-\omega_m\sin^2\theta_k]. \tag{3}$$

In the above, θ_k is the angle between $\mathbf{k}$ and the magnetization direction, ω is the angular excitation frequency, ω_m is the product of the gyromagnetic ratio γ and the saturation induction $4\pi M$, and ω_{res} is the uniform precession frequency. For a given internal field H the relation between k and θ_k is given by the spin-wave dispersion relation

$$\omega_k=\gamma[(H+Dk^2)\,(H+Dk^2+4\pi M\,\sin^2\theta_k)]^{1/2}, \tag{4}$$

with $\omega_k=\omega/2$. In Eq. (4), D is an exchange parameter on the order of 5×10^{-9} Oe-cm² for YIG.

The above equations provide the necessary information to calculate theoretical thresholds. The experimentally observed threshold should correspond to the minimum value of h_c with respect to k at a given value of the static field H and the pump angle ψ. In Ref. 4 the k-dependence of ΔH_k was neglected and the minimization was carried out only on the $|\,W\,|^{-1}$ factor in Eq. (1). In order to demonstrate the effect of the ΔH_k k-dependence

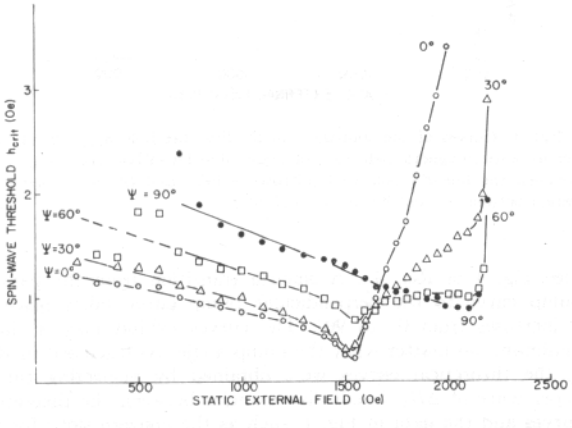

FIG. 2. Curves of h_{crit} at 9.1 GHz versus the static magnetic field for single-crystal YIG and oblique pumping. The pump angle ψ was varied between 0° and 90° in steps of 30°.

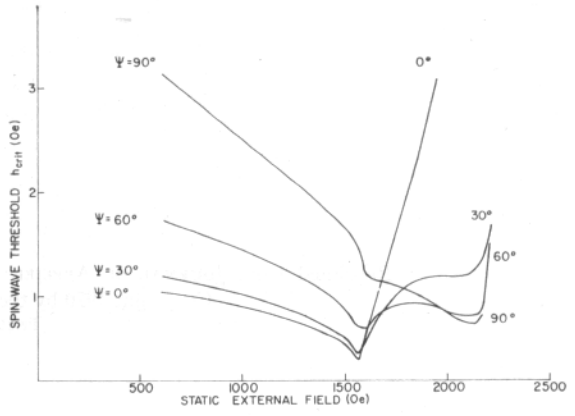

FIG. 3. Theoretical oblique pumping h_{crit} curves based on the $\Delta H_k(\mathbf{k})$ from parallel pumping data on single-crystal YIG at 9.1 GHz and a trial function for ΔH_k as given in the text.

on the threshold, some reasonable trial function for $\Delta H_k(\mathbf{k})$ is required so that the minimization can be carried out on $(\Delta H_k/|\,W\,|)$. For the present evaluation, the trial functions were based on the h_{crit} determinations from parallel pumping. Such data give ΔH_k vs θ_k for $k=0$ and ΔH_k vs k for $\theta_k=\pi/2$. A general θ_k dependence for nonzero k is not accessible. The trial functions were chosen so that ΔH_k reduced to the parallel pump values for $k=0$ or $\theta_k=\pi/2$. Obviously, the resultant theoretical h_{crit} curves fit the data for $\psi=0°$ (parallel pumping) by definition. The theoretical curves for the other pump angles did exhibit the peculiar characteristics in evidence from the data. Calculated butterfly curves based on a trial function

$$\Delta H_k(\text{Oe})=0.2+8.6\times10^{-7}\cdot k+0.375\,\sin^2\theta_k \tag{5}$$

are shown in Fig. 3. The double minimum for $\psi=60°$ is clearly in evidence and the shapes of the curves are in qualitative agreement with the data. The theory does predict a distortion in the 90° curve near the usual parallel pump minimum which is not evident from the data. This effect, as well as the other small discrepancies, are due to the $\sin^2\theta_k$ term in Eq. (5). The actual θ_k contribution to ΔH_k for $k\neq0$ is probably somewhat weaker than specified by Eq. (5). A more general trial function could be used and the various parameters adjusted to fit the data in order to more accurately determine $\Delta H_k(\mathbf{k})$.

From these results it is clear that the oblique pumping theory is consistent with experimental data if the appropriate k-dependence of ΔH_k is included in the minimization procedure to determine h_{crit}. The results also indicate that oblique pumping data can be used to determine the actual $\mathbf{k}$-dependence of ΔH_k for specific materials by utilizing various trial functions for $\Delta H_k(\mathbf{k})$ and adjusting the parameters to obtain a best fit with the data.

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