selves. The results of KK are consistently lower than the others and show a stronger temperature dependence. KK used a different experimental method and obtained the calcium impurity content from the crystal supplier. For the other results impurity content was measured by a conductivity method and its chemical nature is unspecified. However, as mentioned earlier, high accuracy of the impurity-content determination is not crucial for the calculation of g_+. It is interesting to note that although Davidge's specimens were purer than those of KK by at least an order of magnitude, his isoelectric temperatures fall in the same range as KK, and are much higher than ours for a specimen of similar purity. Since all the experiments purport to measure effects of charged dislocations and the differences lie outside the quoted errors, we are left with a real discrepancy.

If, following KK, one takes into account the association between cation vacancies and neighboring divalent-cation impurities by means of Eq. (8) all values of g_+ are raised slightly. The effect is larger for the higher isoelectric temperatures and is negligible at 350°C for specimens of high purity. With this correction, and assuming that the binding energy of vacancy to impurity ion is the same for all specimens, our results agree best with KK.

The temperature dependent term A_+ in Eq. (2) is found to be $\sim$700 by KK after considering association effects. Since A_+A_- is $\sim$500 for NaCl[12] we have $A_+ \gg A_-$. However, Davidage finds $A_+ \sim 8$ and thus $A_+ \ll A_-$. Although the accuracy attainable in this determination is not great, the difference seems to be larger than one would expect.

In conclusion, there appears to be a need for further experiments to resolve the differences found between the results of the several workers.

JOURNAL OF APPLIED PHYSICS VOLUME 40, NUMBER 1 JANUARY 1969

Threshold Microwave-Field Amplitude for the Unstable Growth of Spin Waves under Oblique Pumping

J. J. GREEN* AND C. E. PATTON*
Raytheon Company, Research Division, Waltham, Massachusetts 02154
AND
E. STERN†
Lincoln Laboratory, Massachusetts Institute of Technology, Lexington, Massachusetts 02173
(Received 12 July 1968)

The threshold microwave-field amplitude h_{crit} required for the unstable growth of spin waves has been investigated for oblique pumping with a linearly polarized microwave field applied at an arbitrary angle with respect to the static magnetic field. The experiment was performed on YIG (1% Dy) spheres at 9.2 GHz, using a rectangular TE_{102} cavity and a pulsed magnetron source. Data were obtained at static external fields of 400, 600, 1100, and 1500 Oe. In general, h_{crit} increases with the pump angle. Theoretical expressions for the oblique pumping h_{crit} have been derived for ferromagnetic insulators of ellipsoidal shape and with axial symmetry about the direction of the static magnetic field sufficient to saturate the sample. The threshold field is determined by maximizing $1/h = \cos\psi/h_{||} \pm \sin\psi/h_\perp$ with respect to the angle between the internal field and the spin-wave propagation direction. Here, $h_{||}$ and $h_\perp$ are the parallel and perpendicular pump threshold fields of Schlömann and Suhl. For external fields sufficient to magnetize the spherical samples to saturation, the theoretical predictions are in good agreement with the experimental data. For small fields, the agreement is poor. However, fair agreement is obtained by assuming a transverse demagnetizing factor N_t greater than the value of $\frac{1}{3}$, which is appropriate for saturated spherical samples. The value of N_t for the best theoretical fit depends on the way in which the sample is demagnetized. This result indicates that for samples not magnetized to saturation, the magnetic structure consists of elongated domains, and that the oblique-pumping data can be explained from the theory for saturated systems, assuming demagnetizing factors appropriate to the domain structure.

INTRODUCTION

The parallel and perpendicular pumping of spin waves in insulating ferromagnets has received considerable attention in recent years. Suhl[1] has calculated the threshold microwave-field amplitude required for the unstable growth of spin waves, with the microwave field perpendicular to the static magnetic field for saturated samples of ellipsoidal shape having a rotational axis of symmetry parallel to the static field. Schlömann *et al.*[2] have considered the situation for the microwave

* The portion of this research performed by J. J. Green and C. E. Patton was supported by the Advanced Research Projects Agency of the Department of Defense and was monitored by Rome Air Development Center under Contr. No. F30602-68-C-0005.
† Operated with support from the U.S. Air Force.
[1] H. Suhl, J. Phys. Chem. Solids **1**, 209 (1957); see also Phys. Rev. **101**, 1437 (1956).
[2] E. Schlömann, J. J. Green, and U. Milano, J. Appl. Phys. **31**, 386S (1960); see also F. R. Morgenthaler, J. Appl. Phys. **31**, 95S (1960). Considerations on parallel pumping were first reported by E. Schlömann, Raytheon Tech. Rept. R-48 (1 October 1959), and by F. R. Morgenthaler, thesis, Massachusetts Institute of Technology (1960).

Downloaded 04 Mar 2004 to 129.82.140.198. Redistribution subject to AIP license or copyright, see http://jap.aip.org/jap/copyright.jsp

field parallel to the static field. There has been little work concerning the threshold fields for the unstable growth of spin waves under so-called oblique pumping with a linearly polarized microwave field at an angle $0 \leq \psi \leq 90°$ to the static-field direction. Stern[3] has discussed oblique pumping as related to remanent ferrite-device operation. Kitaev *et al.*[4] have considered the theoretical problem of pumping with an arbitrary pump field, although specific results were shown only for an infinite medium. An experimental and theoretical investigation of oblique pumping is the subject of the present report.

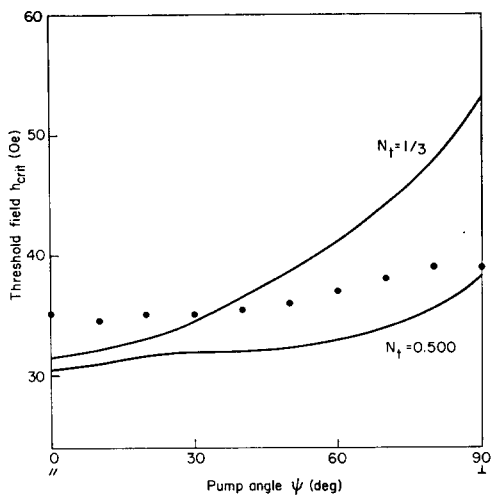

FIG. 1. Experimental and theoretical microwave-field threshold amplitudes for a YIG (1% Dy) sphere at 9.2 GHz as a function of the pump angle between the static magnetic-field direction and microwave-field polarization direction for an external static magnetic field of 400 Oe. The theoretical curves (solid lines) were obtained assuming transverse demagnetizing factors of $\frac{1}{2}$ and $\frac{1}{3}$, as explained in the text.

EXPERIMENT AND RESULTS

The spin-wave instability threshold for oblique pumping was determined from microwave-susceptibility data obtained using cavity-perturbation techniques and a rectangular TE_{102} reflection cavity at 9.2 GHz.[5,6] Determinations of h_{crit} were made as a function of pumping angle ψ for $0° \leq \psi \leq 90°$ for a 1% dysprosium-doped polycrystalline YIG sphere with a linewidth of 50 Oe at 9.2 GHz and a saturation magnetization of 1750 G. This particular sample was chosen for two reasons: (1) The rare-earth contribution to the spin-wave linewidth should dominate over other contributions, thereby facilitating the theoretical interpretation of the data, and (2) the spin-wave linewidth is still small enough to allow measurement of the threshold field h_{crit} with the maximum attainable microwave-field amplitude of 80 Oe.

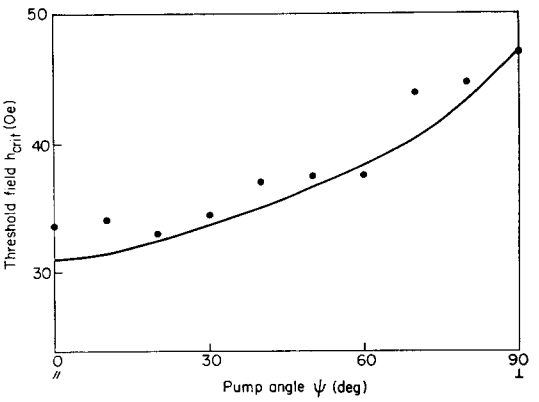

FIG. 2. Experimental and theoretical threshold amplitudes for a YIG (1% Dy) sphere at 9.2 GHz as a function of pump angle for an external static field of 600 Oe. The theoretical curve was obtained with a transverse demagnetizing factor of 0.38.

In Figs. 1–4, the threshold field h_{crit} is shown as a function of pumping angle ψ for external static fields of 400, 600, 1100, and 1500 Oe, respectively. At all but the highest field h_{crit} increases with pumping angle. These data are in qualitative agreement with previous measurements by Green *et al.*[7] which indicated that

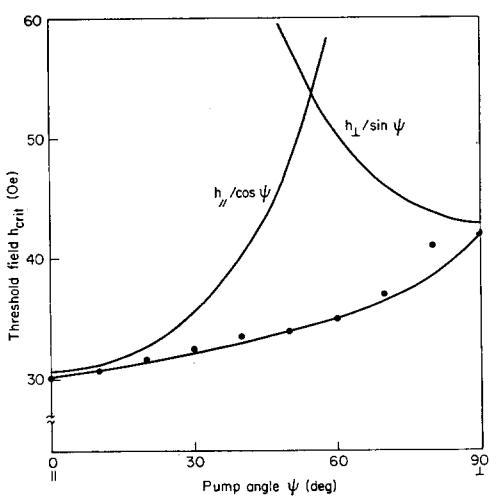

FIG. 3. Experimental and theoretical threshold amplitudes for a YIG (1% Dy) sphere at 9.2 GHz as a function of pump angle for an external static field of 1100 Oe. The lower theoretical curve was obtained by adjusting the dependence of the spin-wave damping on wave vector to give the best fit with a transverse demagnetizing factor of $\frac{1}{3}$. The two upper curves were obtained by considering parallel and perpendicular-pumping processes, which operate independently as explained in the text.

[3] E. Stern, G-MTT Intern. Microwave Symp. Proc., Boston, 1967, IEEE Cat. No. 17C66, p. 103.
[4] L. V. Kitaev, A. A. Goncharova, and G. I. Katamaran, Sov. Phys.—Solid State **9**, 246 (1967).
[5] R. W. Damon, Rev. Mod. Phys. **25**, 239 (1953).
[6] J. J. Green, Sci. Rept. No. 2 (Ser. 2), Gordon McKay Lab., Harvard University, Cambridge, Mass. (1 December 1959).
[7] J. J. Green, J. A. Hillier, and J. H. Saunders, G-MTT Intern. Microwave Symp. Proc., Boston, 1967, IEEE Cat. No. 17C66, p. 100.

Downloaded 04 Mar 2004 to 129.82.140.198. Redistribution subject to AIP license or copyright, see http://jap.aip.org/jap/copyright.jsp

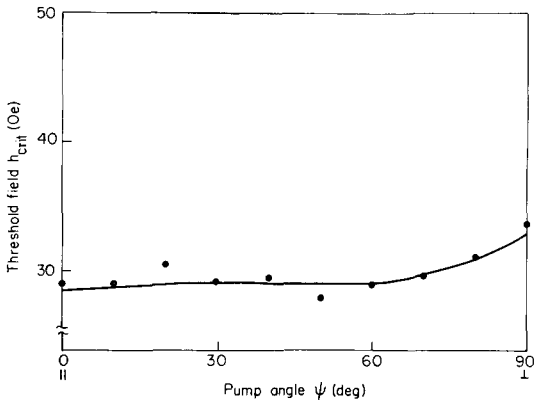

FIG. 4. Experimental and theoretical threshold amplitudes for a YIG (1% Dy) sphere at 9.2 GHz as a function of pump angle for an external static field of 1500 Oe. The theoretical curve was obtained with no adjustable parameters, as explained in the text.

the parallel-pump threshold corresponding to $\psi=0$ was significantly lower than that for $\psi=\pi/2$ and perpendicular pumping. The present data on threshold field as a function of pump angle provide several new conclusions concerning spin-wave instabilities in ferromagnetic insulators. First, these data can be interpreted in terms of the previous calculations for parallel and perpendicular pumping, if the theories are extended to include oblique pumping. Second, the data for samples in a nonsaturated state are also in qualitative agreement with the theory developed for saturated systems, if transverse demagnetizing factors appropriate to the domain shape and not the sample shape are used. In the next section, the theory of oblique pumping will be presented, and in the following section, the above data will be interpreted in terms of the theory.

THEORY

The normal magnetic modes for ferromagnetic samples of ellipsoidal shape have been calculated by Holstein and Primakoff,[8] who obtain a dispersion relation of the form:

$$\omega_k=\gamma[(H_i+Dk^2)(H_i+Dk^2+4\pi M_s\sin^2\theta_k)]^{1/2}, \quad (1)$$

where ω_k is the angular frequency of a spin wave with wave vector of magnitude k and at an angle θ_k with the direction of the internal field H_i, γ is the gyromagnetic ratio (1.76×10^7 Oe^{-1}·sec^{-1}), D is a phenomenological exchange parameter ($\approx5\times10^{-9}$ Oe-cm^2 for YIG), and $4\pi M_s$ is the saturation induction (≈1750 G). In Fig. 5, ω_k is shown as a function of k for $\theta_k=0$ and $\pi/2$, as calculated for YIG from Eq. (1) with an internal field of 180 Oe. The region bounded by these two curves is called the spin-wave manifold.

If a linearly polarized microwave field $he^{i\omega t}$ of frequency ω is applied to the ferromagnetic sample perpendicular to the static internal field (perpendicular pumping), Zeeman, dipolar, and exchange interactions give rise to a coupling between the uniform precession spin wave at ω, driven by the microwave field and other spin-wave states. For sufficiently intense microwave drive, this coupling leads to the unstable growth of certain spin-wave amplitudes. If the microwave field is applied parallel to the static internal field (parallel pumping), the uniform precession amplitude is zero. For parallel pumping, however, spin-wave states are coupled directly to the microwave drive, and spin-wave instabilities may also occur for sufficiently intense microwave-field amplitudes. For the special case of saturated ferromagnetic insulators in the shape of ellipsoids of revolution about the internal-field direction, Suhl[1] has calculated the threshold field $h_\perp$ required for the unstable growth of the spin-wave amplitude for the state (ω_k, k, θ_k) under perpendicular pumping, and Schlömann *et al.*[2] have calculated the corresponding threshold $h_{||}$ for parallel pumping. The calculations indicate that the threshold is particularly low if $\omega_k=\omega/2$, and the pumping of unstable spin waves at $\omega/2$ has been termed a first-order process (strong coupling). For a first-order process, the results are

$$h_\perp=(\omega/\omega_m)(\Delta H_k/\sin\theta_k\cos\theta_k)\times\{[(\omega_0-\omega)^2+(\gamma\Delta H/2)^2]^{1/2}/(\omega/2+\gamma Dk^2+\gamma H_i)\}, \quad (2a)$$

$$h_{||}=(\omega/\omega_m)(\Delta H_k/\sin^2\theta_k), \quad (2b)$$

where ω_m is given by $\gamma 4\pi M_s$, ω_0 is the uniform precession, resonance frequency $\omega_0=\gamma H_i+N_i\omega_m$ for a given internal field H_i, and transverse demagnetizing factor

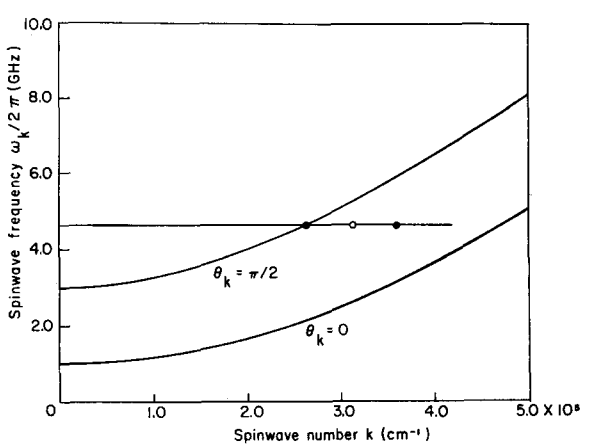

FIG. 5. Spin-wave frequency ω_k as a function of wave-number magnitude k for k parallel to the internal-field direction ($\theta_k=0$) and k perpendicular to the internal field ($\theta_k=\pi/2$). The parallel-pump instability for a spin wave at ω_k occurs at $\theta_k=\pi/2$, and the perpendicular-pump instability occurs at $\theta_k\approx\pi/4$, as indicated by the black dots. The oblique-pump instability corresponds to an intermediate state $\pi/4<\theta_k<\pi/2$, as indicated by the open circle.

[8] T. Holstein and T. Primakoff, Phys. Rev. **58**, 1098 (1940).

Downloaded 04 Mar 2004 to 129.82.140.198. Redistribution subject to AIP license or copyright, see http://jap.aip.org/jap/copyright.jsp

N_t, ΔH_k is the spin-wave linewidth, and ΔH is the uniform precession linewidth at frequency ω. The particular magnon which has the lowest $h_\perp$ for perpendicular pumping or $h_{||}$ for parallel pumping will be the one which goes unstable first and will control the experimentally determined threshold field h_{crit}. For perpendicular pumping, the threshold field corresponds to spin waves with $\theta_k \approx \pi/4$, and for parallel pumping, $\theta_k = \pi/2$ spin waves determine the threshold. These two states are indicated as black dots in Fig. 5 for an excitation frequency of 9.2 GHz. In the above and in the discussion to follow, it is assumed that ω is sufficiently high or H_i is sufficiently low so that $\omega/2$ is above the $\theta = \pi/2$ branch of the manifold at $k = 0$. The theory is strictly valid only for $k > 0$.

In an oblique-pumping experiment, both a parallel microwave-field component $he^{i\omega t}\cos\psi$ and a perpendicular microwave-field component $he^{i\omega t}\sin\psi$ are present, and the possibility exists that some magnon state with $\pi/4 < \theta_k < \pi/2$ determines the threshold field. Such a state is represented by the open circle in Fig. 5. The oblique-pumping threshold field can be calculated in a straightforward manner by considering the hamiltonian for the magnetic system in the presence of an oblique pumping field, calculating the spin-wave amplitude equation of motion, and proceeding according to the technique developed by Suhl and Schlömann. For the present purposes, however, it is more illustrative of the physical processes involved to calculate the threshhold from a circuit analogy, as reported previously by one of the authors.[3] The spin-wave theory is outlined in the Appendix. Assuming linear coupling between the pumping field and the spin system, as illustrated by the equivalent circuit in Fig. 6, and taking the phases of the perpendicular and parallel coupling coefficients $C_\perp$ and $C_{||}$ to be equal, the effective spin-wave drive U may be defined as

$$U = C_{||} h\cos\psi + C_\perp h\sin\psi, \tag{3}$$

where $C_{||}$ and $C_\perp$ depend on the particular magnon state $(\omega/2, k, \theta_k)$ being pumped. In Eq. (3), $h\cos\psi$ and $h\sin\psi$ are the parallel and perpendicular components, respectively, of the microwave-field amplitude. The instability for a particular spin wave occurs when U exceeds an effective drive threshold T, which can be determined by considering the known theoretical threshold for that spin wave from Eqs. (2) for $\psi = 0°$ and $\psi = 90°$. A parallel-pumped instability occurs for $T = C_{||} h_{||}$, and a perpendicular-pumped instability for $T = C_\perp h_\perp$. With $C_{||} = T/h_{||}$ and $C_\perp = T/h_\perp$, Eq. (3) becomes

$$1/h = (T/U)(\cos\psi/h_{||} + \sin\psi/h_\perp).$$

Instability occurs when $U = T$ is satisfied, so that the threshold-field amplitude h_{oblique} required to drive state

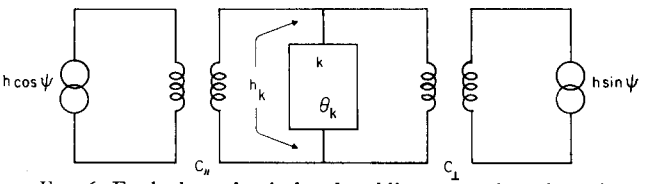

FIG. 6. Equivalent circuit for the oblique pumping of a spin-wave state (ω_k, k, θ_k) with a microwave-field polarization direction at an angle ψ with respect to the static internal field. The parallel- and perpendicular-pumping contributions are characterized by sources $h\cos\psi e^{i\omega t}$ or $h\sin\psi e^{i\omega t}$ and coupling coefficients $C_{||}$ or $C_\perp$, respectively.

$\omega/2, k, \theta_k)$ unstable is given by

$$1/h_{\text{oblique}} = \cos\psi/h_{||} + \sin\psi/h_T. \tag{4}$$

In the Appendix, Eq. (4) is obtained from spin-wave theory. It is shown that the equal-phase assumption can be satisfied by correctly choosing the angle ϕ_k between the component of k in the plane normal to H_0 and the transverse component of the microwave field.

The oblique-pumping threshold field h_{crit} is obtained by maximizing the right-hand side of Eq. (4), thereby minimizing h_{oblique}. If the spin-wave linewidth ΔH_k is independent of k and θ_k, h_{crit} as a function of ψ at constant external field can be calculated from Eqs. (2) and (4). If ΔH_k depends on k and θ_k, the minimization procedure necessary to determine the spin-wave state $(\omega/2, k, \theta_k)$ that gives the lowest threshold h_{crit} is more complicated. An approximate procedure was used to take the k and θ_k dependence of ΔH_k into account.

It was assumed that the k and θ_k dependences of ΔH_k could be expressed in a form

$$\Delta H_k = f_1(k) \cdot f_2(\theta_k) \tag{5}$$

with

$$f_1(k) = A + Bk \tag{6}$$

and

$$f_2(k) = 1 + C(\pi/2 - \theta_k) + F(\pi/2 - \theta_k)^2. \tag{7}$$

Both parallel and perpendicular data indicate that ΔH_k is approximately an increasing linear function of k. The A and B parameters were obtained from ΔH_k vs k determinations on the basis of h_{crit} data for parallel pumping on the Dy-doped material. These results are shown in Fig. 7. The A parameter was determined to be 15.0, and B to be 0.5×10^{-5}. The parameters in $f_2(\theta_k)$ were determined by fitting the theoretical curve of h_{oblique} vs pump angle [calculated from Eqs. (2), (4), and (5)–(7)] to the experimental h_{crit} data for a static external field of 1100 Oe. Using this procedure, C was determined to be 0.15, and F to be -0.3. For a heavy rare-earth doping, ΔH_k is expected to be approximately constant and not exhibit strong k or θ_k dependences. This expectation is satisfied by the above result. The theoretical results would probably not be severely modified if ΔH_k was assumed to be constant. Consequently, the specific form for the weak k and θ_k dependences should not be critical.

Downloaded 04 Mar 2004 to 129.82.140.198. Redistribution subject to AIP license or copyright, see http://jap.aip.org/jap/copyright.jsp

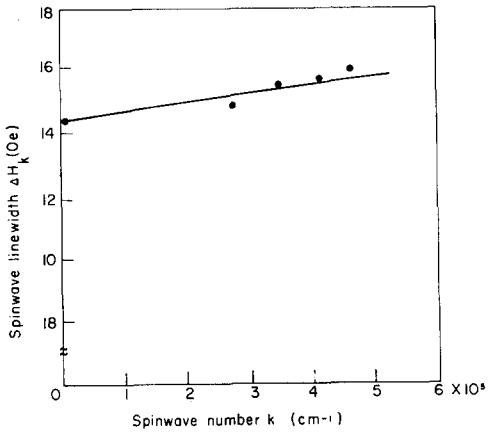

FIG. 7. The experimentally determined k dependence of the spin-wave linewidth ΔH_k for parallel pumping in a 1% Dy-doped YIG sphere at 9.2 GHz.

DISCUSSION

First consider the data for external fields sufficient to magnetize the test sample nearly to saturation, as shown in Fig. 3 for an external field of 1100 Oe, and in Fig. 4 for a field of 1500 Oe. As discussed in the last section, a qualitative best-fit between the theory and the data at 1100 Oe was used to determine the parameters A and B in the θ_k dependence of ΔH_k. As evident from Fig. 3, this two-parameter fit gives good agreement with the data. The theoretical curve was obtained using a transverse demagnetizing factor of $\frac{1}{3}$, appropriate for saturated spherical samples. If oblique pumping simply consisted of simultaneous but independent parallel and perpendicular pumping, one would expect the observed threshold to correspond to either $h_{||}/\cos\psi$ or $h_\perp/\sin\psi$, whichever was lower. The independent pumping model, therefore, predicts an increase in h_{crit} with ψ to about 60°, as indicated by the $h_{||}/\cos\psi$ curve, and a subsequent decrease at higher angles, as indicated by the $h_\perp/\sin\psi$ curve. The agreement between the data in Fig. 3 and the independent pumping model is rather poor. In Fig. 4, the calculated h_{crit} vs ψ curve for a static field of 1500 Oe is shown for comparison with the data. The theoretical curve was obtained using the ΔH_k, θ_k and k dependence, determined from the fit at 1100 Oe. There are no adjustable parameters available to adjust the fit between the data and theory for Fig. 4. The agreement is quite good.

For external fields less than $4\pi M_s/3$, or about 600 Oe for YIG, spherical samples are not magnetized nearly to saturation, and the samples consist of discrete domains. In the presence of domain structure, the theory is not strictly applicable. Nevertheless, it is informative to compare the data for external fields below $4\pi M_s/3$ with the theoretical predictions. Oblique-pumping h_{crit} data as a function of ψ are shown in Figs. 1 and 2 for external fields of 400 and 600 Oe, respectively. First consider the data at 400 Oe. The upper theoretical curve was calculated assuming a transverse demagnetizing factor of $\frac{1}{3}$, appropriate for saturated spheres. The agreement between the data and this theoretical curve is extremely poor. Better agreement can be obtained, however, if the transverse demagnetizing factor is assumed to be $\frac{1}{2}$, as shown by the lower solid curve. With this assumption, the shape of the theoretical curve closely matches the data, although the values are somewhat lower. Some variation in the experimental data has been noted, which depends on the way in which the sample was demagnetized. In general, a good fit to any particular set of data could be obtained with $\frac{1}{3}<N_t<\frac{1}{2}$. Stern[3] has previously reported data on partially magnetized toroids which agree with the theory for $N_t=\frac{1}{2}$. Agreement for $\frac{1}{3}<N_t<\frac{1}{2}$ indicates that the portions of the material which go unstable first correspond to elongated domains. Such domains would have $N_t\approx\frac{1}{2}$. Further support for the proposal that the domain structure is important in determining the instability threshold is provided by the data at 600 Oe, shown in Fig. 2. At 600 Oe, the internal field is approximately zero for YIG, and the material is nearly saturated. The theoretical curve shown in Fig. 2 which provided the best fit to the data was obtained for $N_t=0.38$ (slightly greater than the value of $\frac{1}{3}$ expected for saturated spheres) and was due to the residual-domain structure at 600 Oe. It should be emphasized that the same ΔH_k dependence used to obtain the theoretical curves for external fields greater than $4\pi M_s/3$ was used to calculate the curves shown in Figs. 1 and 2. The only adjustable parameter was N_t. Demagnetization effects due to domain structure have been used to explain large low-field loss in ferromagnetic insulators.[9] The present data and theoretical agreement indicate that domain structure also influences the instability threshold. Demagnetizing factors appropriate to the domain structure and not the sample shape should be used in theoretical estimates of h_{crit} for partially magnetized samples. Any quantitative comparison for systems below saturation, however, will require extension of the theory to include samples which are not ellipsoids of revolution, and the effect of the anti-Larmor component of the linear drive, as well as more precise information concerning domain structure.

SUMMARY

This investigation has provided new information and new understanding concerning the microwave-field thresholds for the excitation of spin-wave instabilities in ferromagnetic insulators. (1) Extensive data on the oblique-pumping threshold as a function of pumping angle have been obtained for external fields, both above and below that required to magnetize the sample to

[9] D Polder and J. Smit, Rev. Mod. Phys. 25, 89 (1953).

Downloaded 04 Mar 2004 to 129.82.140.198. Redistribution subject to AIP license or copyright, see http://jap.aip.org/jap/copyright.jsp

saturation. (2) The theory of parallel and perpendicular pumping in saturated ferromagnetic insulators has been extended to include oblique pumping. (3) The data for saturated samples can be explained by the theory. (4) The data for unsaturated samples can be qualitatively explained by the theory if transverse demagnetizing factors appropriate to the domain structure and not the sample geometry are assumed.

APPENDIX

Following the treatment of Schlömann,[10] the spin-wave instability threshold-field amplitude for oblique pumping can be obtained by considering a Hamiltonian for the magnetic system of the form

$$\mathcal{H}=\sum_k \omega_k b_k b_k^* + \tfrac{1}{2}\sum_{k\neq 0}(f_k b_0^* b_k b_{-k} + \text{c.c.}) - \tfrac{1}{2}\cos\omega t \sum (c_k b_k^* b_{-k}^* + \text{c.c.}), \quad \text{(A1)}$$

where ω_k is the spin-wave frequency, the b_k are spin-wave amplitudes, f_k is a coupling coefficient for perpendicular pumping ($\psi=90°$), and c_k is a coupling coefficient for parallel pumping ($\psi=0°$), assuming a $\cos\omega t$ time dependence for the component of the microwave field along the static field. With parallel and perpendicular microwave-field components $h\cos\omega t\cos\psi$ and $h\cos\omega t\sin\psi$, respectively, the coupling coefficients of Eq. (A1) are[11]

$$f_k=-(\omega_m/2\omega_k)(\gamma H_i+\gamma Dk^2+\omega_k)\exp(-i\phi_k)\sin\theta_k\cos\theta_k \quad \text{(A2a)}$$

and

$$c_k=(\omega_m/2\omega_k)\gamma h\cos\psi\exp(2i\phi_k)\sin^2\theta_k. \quad \text{(A2b)}$$

The perpendicular component of the microwave field is along the x axis, and ϕ_k is measured with respect to this axis. The b_0 in Eq. (A1) is related to the uniform precession amplitude and is given by

$$b_0=[\gamma h\sin\psi/2(\omega_0-\omega)]e^{i\omega t} \quad \text{(A3)}$$

if the anti-Larmor term in the transverse component of the linearly polarized microwave field is neglected. The f_k also neglects the contribution of this term to the coupling. For sufficiently large internal fields, the correction due to these anti-Larmor terms is small and may be neglected.

The first-order instability threshold is obtained from the equation of motion for the b_k,

$$db_k/dt=i\partial\mathcal{H}/\partial b_k^* = i[\Omega_k b_k+(f_k^* b_0-c_k\cos\omega t)b_{-k}^*], \quad \text{(A4)}$$

where $\Omega_k=\omega_k+i\eta_k$. Damping is taken into account by assigning an imaginary part η_k to the spin-wave frequency ω_k. The relaxation rate η_k is related to the spin-wave linewidth by $\eta_k=\frac{1}{2}\gamma\Delta H_k$. The instability threshold is obtained by determining the microwave-field amplitude at which the equation of motion (A4) permits a stationary solution

$$b_k=b_{k0}e^{i\omega t/2} \quad \text{(A5)}$$

in the presence of damping. By considering only terms which vary as $e^{i\omega t/2}$ in db_k/dt and $e^{-i\omega t/2}$ in db_{-k}^*/dt, two homogeneous linear equations in b_{k0} and b_{-k0}^* result. The microwave-field amplitude for which a nontrivial solution to b_{k0} and b_{-k0}^* exists corresponds to the instability threshold. From Eqs. (A4) and (A5), a nontrivial solution for b_{k0} is obtained only for

$$|\Omega_k-\omega/2| = |(c_k/2)-f_k^*\gamma h\sin\psi/2(\Omega_0-\omega)|, \quad \text{(A6)}$$

where the uniform precession frequency ω_0 has been replaced by Ω_0 in the expression for b_0. The relaxation rate η_0 is related to the uniform precession linewidth ΔH by $\eta_0=\frac{1}{2}\gamma\Delta H$. The right-hand side of Eq. (A6) is proportional to the microwave-field amplitude h. Instability occurs for the smallest value of h if the spin-wave frequency ω_k is equal to one-half the pump frequency ω. If $\omega_k=\omega/2$ is satisfied, the critical field is determined by

$$\eta_k=(\omega_m/\omega)(\gamma h/2)\,|\cos\psi\sin^2\theta_k\exp(i\phi_k) + \sin\psi\sin\theta_k\cos\theta_k[(\gamma H_i+\gamma Dk^2+\omega/2)/(\Omega_0-\omega)]|. \quad \text{(A7)}$$

The lowest threshold for a given magnon instability is realized for that value of ϕ_k for which the phases of the two terms inside the vertical bars of Eq. (A7) are equal. The threshold is given by

$$1/h_{\text{oblique}}=(\omega_m\sin^2\theta_k/\omega\Delta H_k)\cos\psi + \frac{\omega_m\sin\theta_k\cos\theta_k(\omega/2+\gamma Dk^2+\gamma H_i)}{\omega\Delta H_k|\Omega_0-\omega|}\sin\psi, \quad \text{(A8)}$$

which is equivalent to Eq. (4).

It is important to realize that $\Omega_0-\omega$ is negative and that the phase of $(\Omega_0-\omega)^{-1}$, which is specified by the relative magnitudes of $\eta_0=\frac{1}{2}\gamma\Delta H$, ω_0, and ω, determines the ϕ_k which leads to the lowest threshold. For narrow linewidth materials, $\omega_0\gg\eta_0$ is satisfied for sufficiently large applied fields, and h_{oblique} corresponds to $\phi_k\approx\pi$. In general, however, the ϕ_k for which Eq. (A8) is valid is given by

$$\phi_k=\pi+\tan^{-1}[\eta/(\omega-\omega_0)]. \quad \text{(A9)}$$

ACKNOWLEDGMENTS

The authors are indebted to E. Schlömann for helpful discussions concerning the theory of oblique pumping, and to J. A. Hillier for making the measurements.

[10] E. Schlömann, Tech. Rept. R-48, Res. Div., Raytheon Co., Waltham, Mass. (October 1959).

[11] The minus sign preceding the expression for f_k was omitted in Eq. (3.46) of Ref. 10.

Downloaded 04 Mar 2004 to 129.82.140.198. Redistribution subject to AIP license or copyright, see http://jap.aip.org/jap/copyright.jsp