General Spin Wave Instability Theory

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***Abstract*—The theory of spin wave instability for magnetic insulators is extended to include generalized anisotropy, ellipsoidal shape, and a microwave pumping field with a general polarization for both first and second order processes. Representative results are given for the instability threshold microwave field amplitude vs. static field and the corresponding critical modes for a Zn–Y hexagonal ferrite thin disk.**

***Index Terms*—Ferrites, magnetic anisotropy, nonlinear magnetics, spin waves.**

I. INTRODUCTION

SPIN WAVE instability is a nonlinear process which occurs in magnetic materials when the applied microwave magnetic field reaches some threshold value. At this point, energy is transferred from the microwave field to parametrically excited spin wave pairs. The particular critical modes which are excited at threshold depend on the details of the sample configuration, the static field, the anisotropy, and the pumping configuration. The initial theory of spin wave instability was developed by Suhl [1] for subsidiary absorption and resonance saturation, and by Schlömann *et al.* [2] for parallel pumping. A cogent review of the full theory is given in [3]. The combination of parallel pumping and subsidiary absorption, termed oblique pumping, was considered in [4], [5]. The effect of anisotropy on instability processes was considered in [4]–[6]. References [4] and [5] presented a generalized tensor approach to anisotropy with specific calculations for the cubic case in the high field limit. In [6] Schlömann *et al.*considered the case of easy plane hexagonal ferrites.

Most of the previous theories were concerned with one special case or another. None of these works provide a fully general theoretical framework for the analysis of first and second order processes in an insulating material of general shape, with a general anisotropy, or for a possibly noncollinear magnetization and field. The purpose of the work described below was to develop such a theory. The formalism includes provision for a sample of ellipsoidal shape, with a general free energy based magnetic anisotropy, and subject to a general microwave pumping field configuration.

This new formulation is based on the formalism of [1] and the general effective field approach given in [5]. The theory considers only spin waves in the short wavelength limit for which sample surface effects can be neglected. The analysis yields working formulae for the threshold microwave field amplitude for spin wave instability, denoted by h_c, as a function of the spin wave decay rate and the spin wave wave vector $\mathbf{k}$. The spin wave decay rate is expressed in linewidth units through an equivalent "spin wave linewidth" parameter ΔH_k. The h_c threshold is $\mathbf{k}$ dependent. For a specific material, sample shape, static magnetic field, and microwave frequency and pumping configuration, one may determine the minimum h_c relative to all of the available spin wave modes and the corresponding critical mode $\mathbf{k}$ for this minimum. This minimum threshold, denoted as h_{crit}, should then correspond to the microwave threshold observed experimentally.

Section II describes the theoretical approach, but without detailed formulae. The full theory and a detailed development of all working equations will be published separately. Section III presents an example of the analysis for a particular case of current technological interest, oblique pumping for first order processes in an easy plane magnetic insulator with parameters which match those for Zn–Y hexagonal ferrite.

II. THEORY

Consider an ellipsoidal-shaped single domain ferrite sample in an externally applied uniform static magnetic field $\mathbf{H}_{\text{ext}}$ and a microwave pumping field $\mathbf{h}$. One starts with the torque equation of motion for the general magnetization vector $\mathbf{M}$ [7],

$$\frac{d\mathbf{M}}{dt} = -|\gamma|\mathbf{M} \times \mathbf{H}_{\text{eff}}, \qquad (1)$$

where $|\gamma|$ is the absolute value of the electron gyromagnetic ratio and $\mathbf{H}_{\text{eff}}$ is the total effective magnetic field. The effective field $\mathbf{H}_{\text{eff}}$ consists of the external static field $\mathbf{H}_{\text{ext}}$, the microwave pumping field $\mathbf{h}$, the demagnetizing field, and effective fields due to anisotropy and exchange. The demagnetizing and anisotropy fields contain both static and dynamic components. These two general fields may be written in the form $\mathbf{N} \cdot 4\pi\mathbf{M}$ and $\mathbf{A} \cdot \mathbf{M}$, respectively, where $\mathbf{N}$ and $\mathbf{A}$ are tensors. The effective exchange field has dynamic components only.

Initially, (1) is developed in a reference frame defined by the principal axes of the ellipsoidal sample. For simplicity, the anisotropy axes may be taken to coincide with these axes as well, with the $\mathbf{A}$ tensor developed accordingly. Alternatively, one may use a completely general anisotropy and develop the $\mathbf{A}$ accordingly. For any given value of the static field $\mathbf{H}_{\text{ext}}$, the static magnetization $\mathbf{M}_{\text{S}}$ will not necessarily be lined up with $\mathbf{H}_{\text{ext}}$. The static effective field, however, will always be parallel to $\mathbf{M}_{\text{S}}$ under conditions of static equilibrium. It is convenient, therefore, to work in a so-called precessional frame in which the z-direction coincides with the direction of $\mathbf{M}_{\text{S}}$. This direction is determined through the usual static equilibrium considerations. One then transforms the magnetization vector and all fields into

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the precessional frame. This automatically yields specific equations for which all static fields are along z. One has dynamic components of $\mathbf{M}$ and $\mathbf{H}_{\rm eff}$ both along z and transverse to z.

Based on the precessional frame formulation, one can now obtain the equations of motion for the spin wave amplitudes. One simply does a plane wave Fourier expansion for the dynamic components of $\mathbf{M}$ in the precessional frame. One then considers specific pairs of terms in this expansion for $\pm\mathbf{k}$, where $\mathbf{k}$ is a general spin wave wave vector. Terms with $\mathbf{k} = 0$ correspond to the uniform mode. Such terms, when present, are connected with components of the microwave field which are transverse to the z-direction.

Through the above procedure, one obtains coupled nonlinear equations for the amplitudes of particular spin waves with wave vectors $\mathbf{k}$ and $-\mathbf{k}$, taken as a_k and a_{-k}. These equations have the form

$$-i\dot{a}_k = (A_k + C_k)a_k + (B_k + D_k)a^*_{-k}. \qquad (2)$$

The A_k and B_k represent linear terms. If one applies the Holstein–Primakoff transformation [7] to the linear terms in (2) only, one obtains the appropriate spin wave dispersion relation for the spin wave frequency ω_k vs. $\mathbf{k}$. If one applies this same transformation to (2) in its entirety, one obtains a new equation of motion of the form

$$-i\dot{b}_k = \omega_k b_k + F_k b_k + G_k b^*_{-k}. \qquad (3)$$

The F_k and G_k coefficients in (3) contains terms which include a_0 factors and components of the internal microwave field in various combinations.

As discussed in [3], the G_k term in (3) is responsible for the spin wave instability. The G_k coefficient increases with the microwave pumping field. Spin wave instability occurs when G_k exceeds the spin wave relaxation rate η_k. The condition $G_k = \eta_k$ determines the spin wave instability threshold h_c for the given spin wave modes at ω_k and $\pm\mathbf{k}$. One then seeks modes out of the ensemble of available spin wave modes with the lowest threshold for the given field, sample parameters, and pumping geometry under consideration. This threshold field is denoted as $h_{\rm crit}$. The ω_k and $\mathbf{k}$ values for this minimum threshold define the critical mode which corresponds to this $h_{\rm crit}$. The spin wave linewidth parameter introduced above is defined by $\Delta H_k = 2\eta_k/|\gamma|$.

The analysis shows that there are two particular spin wave frequencies which can yield the minimum threshold condition, $\omega_k = \omega_p$ and $\omega_k = \omega_p/2$, where φ_k is the pumping frequency. The $\omega_k = \omega_p/2$ condition corresponds to so-called first order processes because the microwave field amplitude h occurs to the first power in the relevant terms in G_k. The $\omega_k = \omega_p$ condition corresponds to second order processes because h occurs to the second power in the relevant G_k terms. Because of the $\omega_k = \omega_p/2$ condition, first order processes give rise to nonlinear effects for static fields which are typically well below the field range for ferromagnetic resonance (FMR). For second order processes with $\omega_k = \omega_p$, the important effects usually occur at fields which are close to the FMR field. If the frequency is low enough, these regions may coincide.

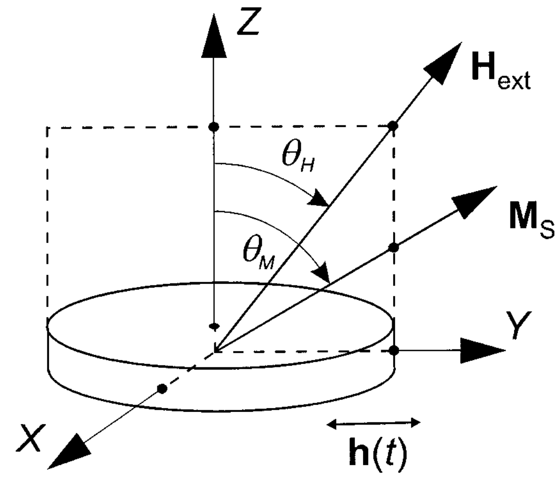

Fig. 1. Sample-field geometry for the Zn–Y disk spin wave instability analysis, with an (X, Y, Z) reference frame with the disk and easy plane in the X–Y plane, the external static magnetic field $\mathbf{H}_{\rm ext}$ in the Y–Z plane at an angle θ_H relative to Z, the static magnetization $\mathbf{M}_{\rm S}$ also in the Y–Z plane at an angle θ_M relative to Z, and the microwave field $\mathbf{h}$ parallel to Y.

The elements of the basic theory given in the above three paragraphs are well established. The specific contribution here is in the inclusion of both a general anisotropy and a general pumping field configuration for both first and second order processes. As noted above, the detailed theory and working equations will be published separately.

III. Sample Results

Consider a thin disk or a film of Zn–Y ferrite with the easy plane in the disk plane. The disk is uniformly magnetized. Fig. 1 shows the sample-field geometry in the sample (X, Y, Z) reference frame with the disk in the X–Y plane. The field $\mathbf{H}_{\rm ext}$ is applied at an angle θ_H relative to the Z-axis and is taken to lie in the Y–Z plane. The static magnetization $\mathbf{M}_{\rm S}$ is also in the Y–Z plane and at an angle θ_M relative to Z. The applied microwave field $\mathbf{h}$ is linearly polarized and along Y. The corresponding right handed precession frame has an x-axis along X, a z-axis along $\mathbf{M}_{\rm S}$, and the y-axis in Y–Z plane. The spin wave vector $\mathbf{k}$ has standard polar and azimuthal propagation angles in the (x, y, z) frame, taken as θ_k and φ_k, respectively.

The results shown below are for the spin wave instability threshold field $h_{\rm crit}$ and the corresponding critical modes as a function of $H_{\rm ext}$ for first order processes only. The control parameter was the magnetic field angle θ_H. The calculations were done for parameters which match realistic experimental situations for Zn–Y, $4\pi M_{\rm S} = 2800$ G, $\omega_p/2\pi = 9$ GHz, $|\gamma| = 2.8$ GHz/kOe, an anisotropy field $H_{\rm A} = 9.0$ kOe, and an exchange stiffness field parameter $D = 5 \times 10^{-9}$ Oe•cm^2/rad^2. The anisotropy field $H_{\rm A}$ is defined by $H_{\rm A} = 2|K_u|/M_{\rm S}$, where K_u is the uniaxial anisotropy energy constant in erg/cm^3.

Fig. 2 shows the calculated spin wave instability threshold field $h_{\rm crit}$ and the corresponding critical mode spin wave wave number k and polar angle θ_k as a function of $H_{\rm ext}$ for first order processes in the easy plane disk at a pumping frequency of 9 GHz. The calculations were done for a full range of the field angle relative to the disk normal, θ_H, from zero degrees to 90°, relative to the disk normal. Specific results in the figure are for

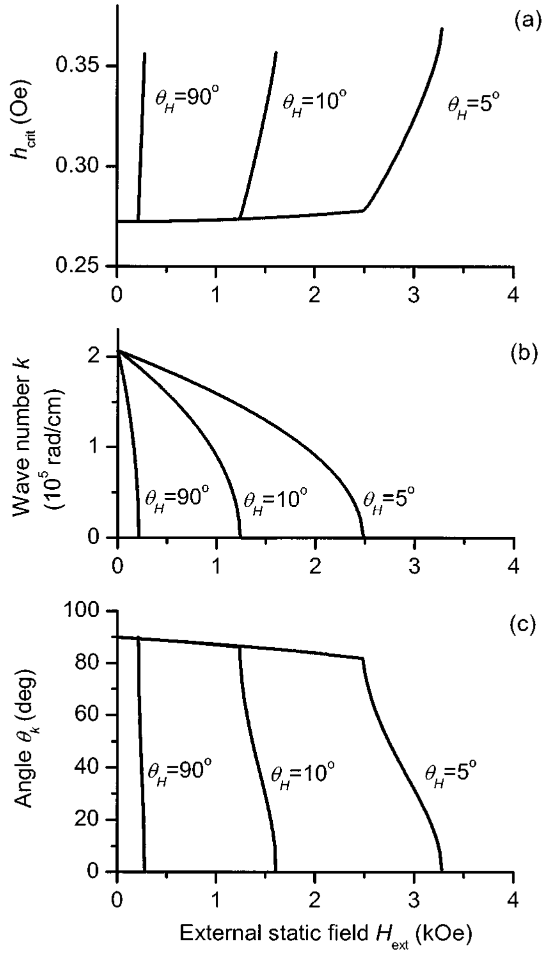

Fig. 2. Threshold and critical mode parameters as a function of the external field H_{ext} for the easy plane disk with 9 GHz microwave excitation. The graphs show (a) first order spin wave instability threshold field h_{crit} vs. H_{ext}, (b) critical mode spin wave wave number k vs. H_{ext}, and (c) polar spin wave propagation angle θ_k vs. H_{ext}. The graphs show results for values of θ_H, the field angle relative to the disk normal, as indicated. The spin wave linewidth ΔH_k was set at 1 Oe for all computations.

θ_H values of 5°, 10°, and 90°. The pumping field $\mathbf{h}$ was parallel to Y in Fig. 1. The spin wave linewidth parameter ΔH_k was set to a constant value of 1 Oe for the evaluations. This means than any effects of a $\mathbf{k}$-dependent spin wave linewidth are ignored.

The results in Fig. 2 are self explanatory, but several points of emphasis are important. 1) The figure does not indicate the values of the azimuthal spin wave propagation angle φ_k for the critical modes. The calculations yielded $\varphi_k = -90°$ over the entire range of $\mathbf{H}_{ext}$ values considered here. This critical mode φ_k value corresponds to the highest ellipticity for the mode and, hence, the lowest threshold. 2) When H_{ext} is very small, all curves in Fig. 2 start from the same values. This limit corresponds to the same physical situation with $\mathbf{M}_S$ along Y. 3) The curves for $\theta_H = 90°$ correspond to parallel pumping and the results are in complete agreement with [6]. 4) There is a clear drop in θ_k and increase in h_{crit} as H_{ext} is increased up to the points where the critical mode k-values become zero. This is due to the gradual change from parallel to oblique pumping as $\mathbf{M}_S$ is pulled away from Y.

The influence of the frequency and the direction of the pumping field on h_{crit} was also investigated. An increase in ω_p leads to the increase of h_{crit}, with graphs which are similar to those in Fig. 2. A change in the direction of $\mathbf{h}$ results in significant changes in h_{crit} and the critical modes. When $\mathbf{h}$ is parallel to Z instead of Y, h_{crit} is about two times greater than in Fig. 2. When $\mathbf{h}$ is parallel to X, h_{crit} is slightly lower than in Fig. 2. This result is surprising. In isotropic ferrites, parallel pumping always yields the lowest h_{crit}.

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